The Institut Mittag-Leffler at Djursholm (Greater Stockholm, Sweden) holds a conference on

Interfaces between Geometric Analysis and Mathematical Physics

from July 4, 2016 to July 8, 2016. Geometric Analysis has emerged as a comprehensive research field over the last 10-15 years. It is determined almost exclusively by

(1) the virtuosity of combining geometric and analytic methods and
(2) its strong motivation by and intimate relation with concepts of classical and modern mathematical physics.

While the first aspect can dominate in everyday work, we shall gather experts in the field to elaborate on the second aspect.
From the technical side, we shall focus on

- *geometric analysis of singular or non-compact spaces*;
- *Kähler geometry and pluripotential theory*; and
- *non-perturbative, in particular topological and conformal foundations of quantum field theory.*

On the conference's focus. To emphasize the conference's focus on *interfaces* and *intelligibility for a mixed audience* and to support the preparation of talks devoted to that aim, we recall the following quotes of two giants of geometric analysis.

> **David Mumford**: "The thing that leaps to mind is something about the suicidal tendency in math to get more and more technical and never to think about explaining one's ideas to mathematicians in other fields of math (let alone other scientists or even the general public). The field has a strange psychology linked to the fear of being thought dumb if you don't know everything."
>
> **Norbert Wiener**: "... there has been a tendency, visible here and there, to give up the search for a great stroke or a great aperçu and to be content with a sort of mathematical embroidery... This reinforces the tendency toward the thin and the bodiless change, which is one of the besetting sins of the pure mathematics of the present time and often burgeons into mountains of triteness and bad taste."

Organizers are Bernhelm Booß-Bavnbek booss@ruc.dk **(Roskilde), Matthias Lesch** lesch@math.uni-bonn.de **(Bonn), George Marinescu** gmarines@math.uni-koeln.de **(Köln), Nicolai Reshetikhin** reshetik@math.berkeley.edu **(Berkeley), and Boris Vertman** vertman@uni-muenster.de **(Münster).**

Program

Format. On Monday morning, after the opening at 9:00 there will be two talks and a guided tour around the estate. All the other morning sessions shall have three talks.
There will be two afternoon talks on Monday and Thursday, three on Tuesday, and none on Wednesday (excursion) and Friday (end of the conference after lunch).
Each talk will be scheduled to 75 minutes, i.e., 60 minutes for the net presentation (including all explanations about concepts, goals, results, challenges etc) and 15 minutes for interruptions, time-outs, questions, further explanations, objections, discussions.

Rough schedule.

07:30-08:45: Breakfast
09:00-10:15: First morning talk, 60 minutes net + 15 minutes discussions
10:15-11:30: Second morning talk, 60 minutes net + 15 minutes discussions
11:30-11:45: Break, water, coffee, fruits
11:45-13:00: Third morning talk, 60 minutes net + 15 minutes discussions
13:00: Lunch
14:30-15:45: First afternoon talk, 60 minutes net + 15 minutes discussions
15:45-16:00: Break, water, coffee, fruits
16:00-17:15: Second afternoon talk, 60 minutes net + 15 minutes discussions
17:15-18:45: Free debate or third afternoon talk, 60 minutes net + 15 minutes discussions

Monday, July 4.

Morning: "Quantum field theories I"

(1) **Giovanni Morchio, Topology and symmetries in quantum mechanics on manifolds**. The basis and alternatives for Quantum Mechanics on Manifolds are discussed. The observable algebra A is obtained from the representations of the diffeomorphism group of M satisfying the Lie-Reinhart module structure of its generators with the (C^∞) functions on the manifold.

(2) **Holger Bech Nielsen, Interpretation of a newly found excess of diphoton decays in LHC as a bound state of 6 top + 6 anti top quarks**. Colin D. Froggatt, myself and other collaborators have long speculated about 6 top quarks + 6 anti top quarks binding so strongly together that they form a bound state or resonance with a mass appreciably lower than the collective mass of 12 top quarks. Now very recently both the ATLAS and CMS experiments at LHC at CERN have found a diphoton excess with mass 750 GeV. We interpret this possibly new particle as being our bound state. The mass fits very well with our principle of multiple point principle, which is the assumption that there exist several versions of vacuum all with almost zero energy densities.

(3) **Guided tour around the estate**

Monday Afternoon, July 4: "Analytic torsion, spectral geometry"

(1) **Jean-Michel Bismut, Hypoelliptic Laplacian and the eta invariant**. Selberg's trace formula is an extension of Poisson formula. In previous work, we have used geometric methods to obtain an explicit formula for the orbital integrals of heat kernels associated with reductive groups of arbitrary rank, very similar to the index formula of Atiyah and Singer. The key ingredient is the construction of a family of hypoelliptic Laplacians, that interpolate in the proper sense between the Casimir and the geodesic flow.
Using methods of harmonic analysis, Moscovici and Stanton had obtained an explicit formula for the eta invariant of a classical Dirac operator of a locally symmetric space in terms of closed geodesics.
In the talk, I will explain how to reobtain the results of Moscovici and Stanton still using geometric methods. Our approach is based on the intimate relation between the Dirac operator of Kostant on a reductive group and the classical Dirac operator on the symmetric space.
From a physics point of view, while the Lagrangian associated with the trace of the standard heat kernel is the energy $E = \frac{1}{2}\int_{S^1} |\dot{x}|^2 ds$, the Lagrangian corresponding to the hypoelliptic deformation is given by $I_b = \frac{1}{2}\int_{S^1}\big(|\dot{x}|^2 + b^4|\ddot{x}|^2\big)ds$. In the case of locally symmetric spaces, the corresponding partition function can be shown not to depend on b, which explains part of our original results on orbital integrals. Our results on eta invariants are just a secondary version of this fact.

(2) **Liviu Nicolaescu, Random Morse functions and spectral geometry**. I will discuss universality results concerning the distribution of critical points and critical values of random series of eigenfunctions of the Laplacian on a compact Riemann manifold.
The distribution of critical points resembles the statistics of random isotropic energy landscapes on Euclidean spaces investigated by Y. Fyodorov about a decade ago in connection with questions in statistical physics. The statistics of critical values is expressed explicitly in terms of the statistics of random symmetric matrices used by Eugene Wigner in the 1950s to explain the slow neutron resonances.
Additionally, I will explain how to reconstruct the geometry of the background manifold from the statistics of a random series of eigenfunctions. This result is a manifestation of the principle: wave propagation is very sensitive to the geometry of the ambient space.

Tuesday, July 5.

Morning: "Singular elliptic calculi"

(1) **Thomas Krainer, The Friedrichs extension for elliptic wedge operators of second order**. Let M_0 be a smooth manifold without boundary, and let Y be a submanifold. Introducing polar coordinates in a tubular neighborhood of Y gives rise to a manifold M with boundary, where the boundary inherits a fibration $\wp : \partial M \to Y$ by spheres. Lifting differential operators from M_0 to M yields operators that are singular on the boundary ∂M. Wedge operators are a generalization of what is obtained by this process and are associated with generic manifolds M

with boundary, where the boundary is fibred by closed manifolds. It is important to note, in the motivating blow-up example, that the class of wedge operators is much broader than what is obtained by merely lifting differential operators from M_0 that are smooth near Y to M: For example, Schrödinger operators on M_0 with potentials that exhibit inverse-square blow-up near Y also give rise to wedge operators on M. The analysis of generic wedge differential operators on manifolds with boundary is at the interface of geometric analysis on stratified spaces and mathematical physics with applications and implications for both fields. Geometric operators associated with incomplete edge geometries on the one hand, and Schrödinger operators with certain singular potentials (such as potentials with inverse-square singularities as previously mentioned) on the other hand, are examples that naturally fit into this unified framework.

In this talk, I plan to discuss a regularity result for elliptic wedge operators of second order that are semibounded from below. More precisely, we describe explicitly the domain of the Friedrichs extension, utilizing standard edge Sobolev spaces on M to capture the part in the minimal domain, and the part beyond the minimal domain is described in terms of the asymptotic behavior of its elements on the boundary with precise regularity statements. This is joint work with Gerardo Mendoza (Temple University, Philadelphia).

(2) **Gerd Grubb, Boundary value problems for fractional-order pseudodifferential operators**. There is currently a great interest in the fractional Laplacian $(-\Delta^a)$ and related pseudodifferential operators, in probability and finance as well as mathematical physics and geometry. The talk will describe recent progress in the analysis of boundary value problems for such operators, which do not satisfy the transmission condition of Boutet de Monvel.

(3) **Elmar Schrohe, The porous medium equation on manifolds with conical singularities**. We consider the porous medium equation

$$\begin{aligned} u'(t) - \Delta(u^m(t)) &= f(u,t), \quad t \in (0, T_0], \\ u(0) &= u_0 \end{aligned}$$

on a manifold with conical singularities. We assume that $m > 0$ and $f = f(\lambda, t)$ is a holomorphic function of λ on a neighborhood of $\mathrm{Ran}(u_0)$ with values in Lipschitz functions in t on $[0, T_0]$.

We model the manifold with conical singularities by an n-dimensional compact manifold $\mathbb{B}$ with boundary, $n \geq 2$, endowed with a degenerate Riemannian metric g, which, in a collar neighborhood $[0,1) \times \partial\mathbb{B}$ of the boundary $\partial\mathbb{B}$, is of the form $g(x,y) = dx^2 + x^2 h(y)$ for a (non-degenerate) Riemannian metric h on $\partial\mathbb{B}$. By Δ we denote the Laplacian associated with this metric; it naturally acts on scales of weighted Mellin-Sobolev spaces $\mathcal{H}_p^{s,\gamma}(\mathbb{B})$.

Given a strictly positive initial value u_0 we show existence, uniqueness and maximal L^p-regularity of a short time solution. In particular, we obtain information on the short time asymptotics of the solution near

the conical point. Our method is based on bounded imaginary powers results and R-sectoriality perturbation techniques.
Actually these solutions turn out to be instantaneously smooth for positive time outside the conic singularity. We obtain long time existence of maximal regularity solutions when the initial data are positive and the forcing term f is zero and show how to establish the existence of weak solutions for the case of non-negative initial data and zero forcing term.
(Joint work with Nikolaos Roidos)

Tuesday Afternoon, July 5: "Singular and non-compact spaces"

(1) **Werner Müller, Geometric scattering theory and autmorphic forms**. Methods of scattering theory, originally developed in mathematical physics, turned out to be very useful in studying the continuous spectrum of geometric differential operators on special classes of non-compact Riemannian manifolds. One important class are locally symmetric spaces of finite volume. The analysis of the continuous spectrum of Laplace type operators on such manifolds is closely related to the theory of automorphic forms. In the rank one case, starting with the work of Faddejev and Lax/Phillips, the application of scattering theory to automorphic forms is well understood. There is a close analogy with quantum scattering on the real line. The higher rank case can be compared with the quantum N-body Schrödinger operaor. In this case the analysis is much more complicated. However, using additional symmetries which are due to the underlying group structure, the multidimensional scattering problems can be reduced to the rank one case. Of particular interest are quotients of Riemannian symmetric spaces by arithmetic groups. The arithmetic nature of the underlying space has important consequences for the structure of the scattering matrices and the distribution of resonances.
In the talk I will report on recent progress and open problems in this subject.

(2) **Jochen Brüning, On the differential topology of Thom-Mather spaces**. We study a class of abstract stratified spaces which comprises, e.g., simplicial complexes, algebraic varieties, and orbit spaces of proper Lie group actions on manifolds. Generally, they may be thought of as compactifications of a manifold by "adding" lower dimensional manifolds.
These spaces are named after René Thom and John Mather and originated with the problem of stability of smooth mappings. As a general reference, we recommend John Mather's article [M] and the forgoing bibliography presented in the (very illuminating) introduction by Mark Goresky (to the same issue of the Bulletin AMS). It is of interest to develop differential geometry and elliptic theory on such spaces which presupposes, however, a certain amount of differential topology. In this talk, we will concentrate on an extension of the Ehresmann Theorem and the Whitney Embedding Theorem (into $\mathbb{R}^m$) to Thom-Mather spaces. We will sketch the proofs and outline some (possible) applications to

elliptic theory, in particular the spectral and index theory of Dirac operators.

Thom-Mather spaces are of increasing interest in modern mathematical physics. They arise in the first place as typical moduli spaces, e.g. of pointed Riemann surfaces or magnetic monopoles. In quantum field theory, they appear as spaces to which Feynman integrands can be extended and computed, as in [AS]. Another, more recent interest derives from new methods for computing amplitudes, e.g. as the volume of the "amplituhedron" introduced by Arkani-Hamed, Hodges and Trnka [AhHT], also known as the positive Grassmannian, a stratified subspace of a real Grassmannian. This approach promises to provide a more intuitive, geometric basis for amplitude calculations.

References

[ArHT] N. Arkani-Hamed, A. Hodges, J. Trnka: *Positive Amplitudes In The Amplituhedron.* arXiv:1412.8478v1[hep-th].

[AS] S. Axelrod, I.M. Singer: *Chern-Simons Perturbation Theory.II.* J. Diff. Geom.**39** (1994), 173–213.

[M] J. Mather: *Notes on Topological Stability.* Bull. AMS **49** (2012), 475-506.

(3) **Richard Melrose, Adiabatic limits and non-compact spaces**. I will give an overview of the appearance of semiclassical limits, and their generalization as (multiple) adiabatic limtis, in problems arising from non-compact spaces and transitions. In particular this includes examples of the compactification of non-compact spaces, such as the Riemann moduli space and planar Hilbert schemes and of gluing problems for Kähler metrics.

Wednesday, July 6.

Morning: "Kähler geometry"

(1) **Semyon Klevtsov, Geometry and large N limits in Laughlin states**. Laughlin states explain the fractional Quantum Hall effect (QHE), when quantized Hall conductance is given by a simple fraction $\sigma_H = 1/\beta, \beta \in Z_+$. Considering Laughlin states on Riemann surfaces allows us to elucidate novel quantized coefficients in QHE, hidden for the planar geometry. Mathematically, this problem makes use of methods in Kähler geometry, local index theorems, geometric analysis on singular Riemann surfaces, as well as $2d$ quantum field theory.

The key object of our study is the partition function for the integer QHE and for the Laughlin states on Riemann surface with arbitrary metric and complex structure and inhomogeneous magnetic field. We determine its asymptotics in the regime of large number of particles using Bergman kernel methods (for IQHE) and path integral methods (for Laughlin states). We then study adiabatic geometric transport on the moduli space, making use of Quillen-Bismut-Gillet-Soule theory (in IQHE case), and also compute a novel transport coefficient for the Laughlin states, arising on higher genus surfaces. Using local index theorem we show that the adiabatic phase acquired by the wave function under adiabatic transport is given by the gauge and gravitational Chern-Simons functional. Finally, we study IQHE and Laughlin states on singular surfaces. The partition function for IQHE states is controlled by the determinant of the Laplacian on singular surface, for which explicit formulae can be easily obtained in specific cases. For Laughlin state, the partition function is conjecturally related to correlation functions in Liouville quantum gravity.

(2) **Xiaonan Ma, Quantum Hall effect and Fekete points**. Let L be a line on a smooth Riemann surface. In the study of the Quantum Hall Effect (QHE) on a Riemann surface we consider the wave functions given by the holomorphic sections of the p-tensor powers of a positive line bundle. The partition function is the square of the L^2-norm of the Slater determinant built with the help of a basis of such sections. We will explain some mathematical problems related to QHE. In particular, we will consider Fekete configurations, which are systems of points maximizing the pointwise norm of the Slater determinant. We will explain the convergence speed of the probability measure of the Fekete configurations, this result holds for any projective manifold.

(3) **Zbigniew Blocki, Geodesics in the space of Kähler metrics and volume forms**. We discuss optimal regularity of geodesics in the space of Kähler metrics of a compact Kähler manifolds, as well as the space of volume forms on a compact Riemannian manifold. They are solutions of nonlinear degenerate elliptic equations: homogeneous complex Monge-Ampère equation and Nahm's equation (introduced by Donaldson), respectively. The highest regularity one can expect is $C^{1,1}$.
For a Riemann surface the two cases coincide and, as noticed by Donaldson, they can be viewed as a generalization of Nahm's equations from a matrix group to the infinite dimensional space of Hamiltonian diffeomorphism of the surface. The original Nahm's equations are equivalent to finding a particle path in a Riemannian symmetric space, subject to a potential field, with prescribed end points.

Wednesday Afternoon, July 6: Excursion

Thursday, July 7.

Morning: "Bifurcations, periodic orbits, and characteristic numbers"

(1) **Nils Waterstraat, Spectral flow and bifurcation**. Bifurcation theory attempts to explain various phenomena that have been discovered and described in the natural sciences over the centuries. Classical examples are, e.g., the buckling of the Euler rod and the appearance of Taylor vortices. In recent years many applications to quantum physics have been found, in particular, to a number of theoretical examples which are difficult to access experimentally.
Mathematically speaking, bifurcation theory is a field of nonlinear analysis that studies branches of solutions of equations of the type $F(\lambda, x) = 0$, where $F : [0,1] \times X \to Y$ is a continuous map and X, Y are real Banach spaces. The aim of this talk is to show how concepts of global analysis, like the spectral flow, can be used to investigate bifurcation phenomena. Moreover, I will also report on ongoing work on

multiparameter bifurcation theory, where maps $F : \Lambda \times X \to Y$ are considered for arbitrary compact topological spaces Λ, and bifurcation invariants are obtained in the KO-theory of the parameter space.

(2) **Paolo Piccione, A Morse theoretical proof of a multiplicity result for periodic orbits of Lagrangian systems**. I will discuss a finite dimensional variational scheme for orthogonal geodesic chords in a compact Riemannian manifold with boundary. As an application, I will present a proof of the existence of multiple brake orbits in a potential well of a conservative Lagrangian system. A similar finite dimensional setup is also available in a general relativistic spacetime, where the arrival time functional characterizes light rays between extended sources and extended receivers. In this case, I will discuss how to use this setup to study the singularities of the exponential map around a degenerate light ray. The results presented are part of a joint work with R. Giambo and F. Giannoni.

(3) **Ronny Thomale, Perfect fractional Chern insulators**. Fractional quantum Hall effect (FQHE) has been a vibrant field of experimental and theoretical condensed matter physics that allowed to reach out into questions of mathematical physics. Within a recently emerged subfield thereof, efforts are taken to realize fractional quantum Hall systems in lattice band structures with non-trivial Chern numbers, referred to as fractional Chern insulators (FCIs). There, complex manifolds, and in particular the intricacies of the Fubini Study metric and the Berry curvature, are vital to forming "perfect" FCIs in the sense of optimal Berry curvature homogeneity, flat lattice dispersion, and overall similarity to the usual continuum FQHE setup. I will communicate a physicists perspective on the current developments along these directions.

Thursday Afternoon, July 7: "K-theory and non-commutative geometry"

(1) **Paolo Piazza, Eta and rho invariants of Dirac operators**. Eta invariants of Dirac operators have been widely used both in Mathematics and Theoretical Physics. In this lecture I will first talk about eta invariants in geometric situations that go beyond the case originally treated by Atiyah, Patodi and Singer. For example foliations and stratified pseudo manifolds. I will then move on to rho invariants and their use in "distinguishing structures".
Rho invariants are, in general, numeric (von Neumann) invariants; under additional hypothesis they can be defined in the K-theory of suitable C^*-algebras. I will explain both the von Neumann and the K-theoretic point of view and illustrate how to use these invariants in order to prove purely geometric theorems. It is an interesting question, very appropriate for this workshop, as to whether rho invariants can be effectively used in physics.
Part of this talk is based on collaborations with Moulay Benameur, Thomas Schick and Boris Vertman.

(2) **Ryszard Nest, Non-commutative geometry perspective towards quantum gravity**. We will sketch some perspectives towards a theory of gravity, both classical and quantum, that come out of the paradigm of Non-commutative Geometry. A couple of examples is the

ongoing work on standard model via notion of spectral action that as a byproduct produces Einstein-Hilbert action and the work on quantum holonomy theory as a candidate for a non-perturbative theory of quantum gravity coupled to fermions.

(3) **Conference Dinner, 19:00**

Friday, July 8.

Morning: "Spectral geometry and quantum field theories II"

(1) **Ksenia Fedosova, The analytic torsion of finite volume orbifolds**. The main objects of this talk are hyperbolic orbifolds, i.e. quotients of a hyperbolic space by discrete groups, which possibly contain elements of finite order. To every finite-dimensional representation of such a group is associated an orbibundle. We consider the varying orbibundle over a fixed orbifold. The main question of the talk is: how does the analytic torsion of the orbibundle change and what kind of geometric information about the orbifold can we obtain from it? We also discuss possible applications of the proposed answer to the study of the cohomology of arithmetic groups.
Our main method is the Selberg trace formula. We show that though its geometric side cannot be used to calculate the analytic torsion for a given orbibundle explicitly, it suits well for studying its asymptotic behavior.

(2) **Pavel Mnev, Perturbative BV theories with Segal-like gluing**. We will give an overview of the cohomolohical symplectic (BV-BFV) approach to perturbative quantization of gauge theories on manifolds with boundary. We present explicit examples where partition functions constructed within the BV-BFV framework combine the features of Atiyah-Segal partition functions (compatibility with gluing/cutting of manifolds) with features of effective Batalin-Vilkovisky actions (they satisfy a version of quantum master equation) and have a version of Wilsonian renormalization flow built into them. Partition functions are expressed in terms of the R-torsion and configuration space integrals and in some cases admit a combinatorial (cellular) presentation.
This is a report on a joint work with Alberto S. Cattaneo and Nicolai Reshetikhin.

(3) **Nicolai Reshetikhin, Quantum field theory on a space-time with boundary**. Because quantum field theory has its origins in scattering experiments, the possibility a space time having a boundary was not it the picture until the emergence of conformal and topological field theory. This talk will focus on some recent developments in quantum field theories on space time with boundary.

Interfaces between Geometric Analysis and Mathematical Physics

Institut Mittag-Leffler at Djursholm, July 4-8, 2016, Schedule

	Monday	*Tuesday*	*Wednesday*	*Thursday*	*Friday*
7:30-8:45	Breakfast	Brkf	Brkf	Brkf	Brkf
	Quantum field theories I *Chair: B. Booß-Bavnbek*	*Singular elliptic calculi* *Chair: B. Vertman*	*Kähler geometry* *Chair: G. Marinescu*	*Bifurcations, periodic orbits, charact. numbers* *Chair: L. Nicolaescu*	*Spectral geometry and quantum field theories II* *Chair: G. Morchio*
9:00-10:00	Opening, **Gianni Morchio**	**Thomas Krainer**	**Semyon Klevtsov**	**Nils Waterstraat**	**Ksenia Fedosova**
10:00-10:15	Discussion	Dscs	Dscs	Dscs	Dscs
10:15-11:15	**Holger B. Nielsen**	**Gerd Grubb**	**Xiaonan Ma**	**Paolo Piccione**	**Pavel Mnev**
11:15-11:30	Dscs	Dscs	Dscs	Dscs	Dscs
11:30-11:45	Break	Break	Break	Break	Break
11:45-12:45	Guided tour	**Elmar Schrohe**	**Zbigniew Blocki**	**Ronny Thomale**	**Nicolai Reshetikhin**
12:45-13:00	Dscs	Dscs	Dscs	Dscs	Dscs
13:00	Lunch	Lunch	Lunch	Lunch	Lunch
	Analytic torsion, spectral geometry *Chair: M. Lesch*	*Singular and non-compact spaces* *Chair: P. Piazza*		*K-theory and non-commutative geometry* *Chair: E. Schrohe*	
14:30-15:30	**Jean-Michel Bismut**	**Werner Müller**	Excursion	**Paolo Piazza**	End of Conference
15:30-15:45	Dscs	Dscs	ditto	Dscs	
15:45-16:00	Break	Break	ditto	Break	
16:00-17:00	**Liviu Nicolaescu**	**Jochen Brüning**	ditto	**Ryszard Nest**	
17:00-17:15	Dscs	Dscs	ditto	Dscs	
17:15-18:15	Debate	**Richard Melrose**	ditto	Debate	
18:15-18:30	ditto	Dscs	ditto	ditto	
19:00				Conference Dinner	