ABSTRACTS

Andrei Martinez-Finkelshtein, Baylor University, USA

TITLE: Asymptotics of multiple orthogonal polynomials for cubic weight

ABSTRACT: We consider the type I and type II multiple orthogonal polynomials (MOPs), satisfying non-hermitian orthogonality with respect to the weight $\exp(-z^3)$ on two unbounded contours on the complex plane. Under the assumption that the orthogonality conditions are distributed with a fixed proportion α, we ?nd the detailed (rescaled) asymptotics of these MOPs, and describe the phase transitions of this limit behavior as a function of α. This description is given in terms of the vector critical measure, the saddle point of the energy functional comprising both attracting and repelling forces. These critical measures are characterized by a cubic equation (spectral curve), and their components live on trajectories of a canonical quadratic differential on the Riemann surface of this equation. The structure of these trajectories and their deformations as function of α plays the crucial role. We illustrate our findings with results of several numerical experiments, explain the computational methodology, and formulate some conjectures and empirical observations based on these experiments.

This is a joint work with Gulherme L. Silva (University of Michigan, Ann Arbor).

Mohan Ravichandran, Mimar Sinan University, Turkey

TITLE : A quantitative Gauss-Lucas theorem and an analytical Lieb-Sokal lemma

ABSTRACT: I will show two elementary consequences of the log barrier techniques developed by Marcus-Spielman-Srivastava. The first is a result on how the convex hulls of roots of univariate complex polynomials shrink under repeated differentiation, which yields a quantitative version of the classical Gauss-Lucas theorem. The second is a general result on how the hyperbolicity cone of a real stable polynomial shifts when acted upon a differential operator whose symbol is a multi-affine real stable polynomial. This operation is known to preserve real stability by the Lieb-Sokal lemma and the above mentioned result yields a quantitative version of this result. I'll also discuss extensions and applications.

Myrto Manolaki, University of South Florida, USA

TITLE: Overconvergence and universality of optimal polynomial approximants

ABSTRACT: Given a function f in a Hilbert space of analytic functions on the unit disc, a polynomial p_n is called an optimal polynomial approximant of degree n to $1/f$ if p_n minimizes the norm $\|pf - 1\|$ over all polynomials p of degree at most n. Such optimal polynomial approximants arise naturally in connection with the study of cyclicity in certain weighted Hardy spaces. In this talk, we will discuss some results about the limiting behaviour of (p_n) on subsets of the unit circle. (Joint work with Catherine Bénéteau and Daniel Seco.)

Petter Bränden, KTH, Sweden

TITLE: Quantitative Pólya-Schur theorems

ABSTRACT: A multivariate polynomial is stable if it is non-vanishing whenever all variables have positive imaginary parts. Linear operators preserving stability have been characterized. However recently it has been desired to know quantitatively how the zeros (or the hyperbolicity cone) are deformed under the transformation. Indeed, this is an essential part of the recent solution to the Kadison-Singer problem (via Weaver's KS_r conjecture).

We will talk about two theorems. One which gives quantitative information for any given stability preserver. The other provides better bounds for Weaver's KS_r conjecture for all $r > 2$.

Parts of this talk is based on joint work with Adam Marcus (Princeton).

Tamas Forgacs, Fresno State Univeristy, USA

TITLE: On Generating relations for hyperbolic polynomials and open problems

ABSTRACT: There are recent results (2016, 2017) concerning the location of zeros of a family of polynomials generated by a rational function $\frac{1}{G(z,t)}$. If $G(z,t) = P(t) + zt^r$, where the zeros of P are positive and real then $G(z,t)$ generates a sequence of polynomials $\{H_m(z)\}_{m=0}^{\infty}$, whose terms are eventually hyperbolic. We will discuss some of the current work attempting to draw similar conclusions for other generating functions, and describe open problems that arose during our investigations.

Olga Katkova, Wheelock College, USA

TITLE: Linear finite difference operators with constant coefficients and distribution of zeros of polynomials

ABSTRACT: We provide the full description of linear finite difference operators preserving the class of hyperbolic polynomials and having the form

$$T(P)(x) = \sum_{j=l}^{m} a_j P(x - j\lambda),$$

where $l, m \in \mathbb{Z}$, $l < m$, $a_j \in \mathbb{C}$, $l \leq j \leq m$, $a_l \neq 0$, $\lambda \in \mathbb{C} \setminus \{0\}$. It turns out that this class of operators preserves the set of real polynomials with all zeros in a fixed strip $\{z : \ |\Im z| \leq b\}$ $(b > 0)$. Given a hyperbolic polynomial P some properties of roots of its image $T(P)$ will be discussed. We also describe linear finite difference operators with non-constant coefficients of the form

$$\Delta_{M_1,M_2,h}(f) = M_1(z)f(z+h) + M_2(z)f(z-h),$$

where M_1 and M_2 are complex functions and h is non-zero complex number, such that $\Delta_{M_1,M_2,h}$ preserves the Laguerre-Pólya class $\mathcal{L} - \mathcal{P}I$.

This is a joint work with M. Tyaglov and A. Vishnyakova.

Anna Vishnyakova, Kharkov University, Ukraine

TITLE: On the conditions for entire functions related to the partial theta function to belong to the Laguerre-Pólya class

ABSTRACT: The entire function $g_a(z) := \sum_{j=0}^{\infty} \frac{z^j}{a^{j^2}},\ a > 1$, is called the partial theta function. It is known that there is a constant q_∞ (≈ 3.2336), such that the function g_a (and all its odd Taylor sections $S_{2n+1}(z,a) := \sum_{j=0}^{2n+1} \frac{z^j}{a^{j^2}}$) belongs to the Laguerre-Pólya class if and only if $a^2 \geq q_\infty$. We will discuss the property of belonging to the Laguerre-Pólya class for some special entire functions related to the partial theta function. In particular, we will present the following results obtained jointly with Thu Hien Nguyen. For an entire function $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $a_k > 0$, we show that f belongs to the Laguerre-Pólya class if the quotients $\frac{a_{n-1}^2}{a_{n-2}a_n}$ are decreasing in n, and $\lim\limits_{n\to\infty} \frac{a_{n-1}^2}{a_{n-2}a_n} \geq q_\infty$. We also prove that if the quotients $\frac{a_{n-1}^2}{a_{n-2}a_n}$ are increasing in n and $\lim\limits_{n\to\infty} \frac{a_{n-1}^2}{a_{n-2}a_n} < q_\infty$, then the function f does not belong to the Laguerre-Pólya class.

Jonathan D Leake, UC Berkeley, USA

TITLE: Capacity Preserving Operators

ABSTRACT: The theory of stable polynomials has found applications in many fields, particularly in combinatorics. In the mid 2000s, Gurvits used his notion of the capacity of a stable polynomial to give simpler proofs of the van der Waerden permanent bound and Schrijver's bound on perfect matchings of regular bipartite graphs. To do this, he analyzed the capacity preservation properties of the partial derivative. In this talk, we combine Gurvits' approach with ideas from the Borcea-Brändén characterization of stability preservers to give tight bounds on a large class of capacity preserving operators. We then use this result to give a straightforward proof of Csikvri's (2014) lower bounds on the coefficients of the matching polynomial for biregular bipartite graphs. As a corollary, we obtain another proof of (what used to be called) Friedland's lower matching conjecture.

Andrzej Piotrowski, University of Alaska Southeast, USA

TITLE: Nonreal zero decreasing operators related to orthogonal polynomials

ABSTRACT: Let L be a linear operator on the vector space of real polynomials and let $Z_C(f)$ denote the number of nonreal zeros of the function f. If L has the property that $Z_C(L(p)) \leq Z_C(p)$ for every real polynomial p, then L is called a *complex zero decreasing operator* or, for brevity, a CZDO. In its simplest form, a classical theorem due to Edmond Laguerre shows that the differential operator $xD + a$, where $a \geq 0$, is a CZDO. In a paper from 2015, we generalized Laguerre's theorem by proving that the differential operator $q(x)D + aq'(x)$, where $a \geq 0$, is a CZDO. We also used this generalization to prove a number of results about the so-called *complex zero decreasing sequences* for polynomials expanded in terms

of classical orthogonal polynomials. This talk will survey the main results of the paper and highlight several interesting open problems.

Keywords: complex zero decreasing sequences, diagonalizable linear operators, zeros of polynomials, orthogonal polynomials

Mikhail Tyaglov, Shanghai Jiao Tong University, China

TITLE: Root location of polynomials with totally nonnegative Hurwitz matrix

ABSTRACT: For a given real polynomial

$$p(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n, \qquad a_0 > 0,$$

the $n \times n$ matrix $H_n(p) = (a_{2j-i})$ is called finite Hurwitz matrix, and the matrix $\mathcal{H}_\infty(p) = (a_{2j-i})_{i,j\in\mathbb{Z}}$ is the *infinite* Hurwitz matrix. It is known [2, 3] that the total positivity of the matrix $\mathcal{H}_\infty(p)$ is equivalent to stability of the polynomial $p(z)$ (roots in the open left half-plane), while the totally nonnegativity of [1, 4] the finite Hurwitz matrix $H_n(p)$ does not imply stability of $p(z)$. In this talk, we completely describe root location of the polynomial $p(z)$ whose finite Hurwitz matrix $H_n(p)$ is totally nonnegative.

Nikos Stylianopoulos, University of Cyprus, Cyprus

TITLE: Since Approximation Theory is already there... Bring Potential Theory to Operator Theory!

ABSTRACT: The purpose of the talk is to discuss, by presenting a number of concrete examples, the luck of application of Potential Theory tools in Operator Theory. More precisely, results from the theory of subharmonic functions (for example, the maximum principle, the principle of descent, the lower envelope theorem, the unicity theorem, etc.) have been used extensively in order to study the behavior of extremal, or orthogonal polynomials, in Approximation Theory. Such results, as powerful as they are, have not find their way to the study of spectral properties of certain important operators, given that the eigenvalues of finite central truncations of these operators are, simply, the zeros of associated orthogonal polynomials.

Alex Kreinin, IBM Software, Toronto, Canada

TITLE: Laplace polynomials, Laplace continued fraction and Ramanujan's identity.

ABSTRACT: We introduce Laplace polynomials, $P_k(t)$ and $Q_k(t)$ that form rational approximation to the continued fraction

$$R(t) = \cfrac{1}{t + \cfrac{1}{t + \cfrac{2}{t + \cfrac{3}{t + \cfrac{4}{\ddots}}}}}.$$

studied by Laplace in connection with some mechanical problems.

We compute the generating functions for these polynomials and prove the following seemingly unrelated identities,

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{j=0}^{n} \frac{(n+m)!}{m!j!(m+2n+1-j)!}\, 2^{-n} = \sqrt{2\pi}e^2(\Phi(2) - \Phi(1)),$$

$$\sum_{k=0}^{n}(-1)^k \frac{1}{2k+1}\binom{n}{k} = \frac{2^{2n}(n!)^2}{(2n+1)!}$$

and

$$\cfrac{1}{1 + \cfrac{1}{1 + \cfrac{2}{1 + \cfrac{3}{\ddots}}}} = \sqrt{\frac{e\pi}{2}} - \sum_{n=0}^{\infty} \frac{1}{(2n+1)!!}.$$

We show that these identities actually have common combinatorial nature linked to the continued fraction for the Mills' ratio,

$$R(t) = \frac{\bar{\Phi}(t)}{\varphi(t)},$$

where $\varphi(t)$ is the standard Normal density and $\bar{\Phi}(t) = \int_t^{\infty} \varphi(s)\mathrm{d}s$ is the tail of the Normal distribution.

Peter Dragnev, Purdue University Fort Wayne, USA (Joint work with P. Boyvalenkov - Bulgarian Academy of Sciences, D. Hardin, Ed Saff - Vanderbilt University, M. Stoyanova- Sofia University)

TITLE: Applications of orthogonal polynomials to minimal energy in polynomial metric spaces

ABSTRACT: In this talk we shall exhibit the applications of various orthogonal polynomials (Gegenbauer, Jacobi, Krawtchouk, Hahn) to energy minimization of point configurations in the general settings of polynomial metric spaces (unit spheres, projective spaces, Hamming spaces, Johnson spaces respectively). Namely, we derive universal lower bounds on the energy of such configurations based on the Delsarte-Yudin linear programming method. Techniques for finding the solution of

the LP problem utilize Radau-Lobato quadratures introduced by Levenshtein and Hermite interpolation.

Björn Gustafsson, KTH, Sweden

TITLE: Zeros of orthogonal polynomials related to hyponormal operators

ABSTRACT: In recent joint work with Mihai Putinar and Nikos Stylianopoulos we have investigated zeros of orthogonal polynomials arising in the theory of hyponormal operators. They can equivalently be described as spectra of truncations of such operators. We have several numerical results which show that the zeros in general go deeper into the domain (the interior of the spectrum of the operator in question) compared to what is the case for the Bergman polynomials. However, the theoretical understanding is still poor, only in a few cases we can prove what we see in the pictures.

I will try to explain a little of what we know. See also our recent monograph "Hyponormal quantization of planar domains", Springer Lecture Notes 2199.

Vl. Kostov, Université de Côte d'Azur, France

TITLE: Descartes' rule of signs and sign patterns

ABSTRACT: By Descartes' rule of signs, a real degree d polynomial P with all non-vanishing coefficients, with c sign changes and p sign preservations in the sequence of its coefficients ($c+p=d$) has $pos \leq c$ positive and $neg \leq p$ negative roots, where $pos \equiv c (\mod 2)$ and $neg \equiv p (\mod 2)$. For $1 \leq d \leq 3$, for every possible choice of the sequence of signs of coefficients of P (called sign pattern) and for every pair (pos, neg) satisfying these conditions there exists a polynomial P with exactly pos positive and exactly neg negative roots (all of them simple). For $d \geq 4$ this is not so. For $4 \leq d \leq 8$, in all nonrealizable cases either $pos = 0$ or $neg = 0$.

It was conjectured that this is the case for any $d \geq 4$. We show a counterexample to this conjecture for $d = 11$. Namely, we prove that for the sign pattern $(+,-,-,-,-,-,+,+,+,+,+,-)$ and the pair $(1,8)$ there exists no polynomial with 1 positive, 8 negative simple roots and a complex conjugate pair.

Yacin Ameur, Lund university, Sweden

TITLE: The random normal matrix model: insertion of a point charge

ABSTRACT: I will discuss propertiesof a two-dimensional eigenvalue ensemble with a conical singularity arising from aninserted charge, located in the bulk of the support of eigenvalues. Joint with Kang and Seo.

Patrick Ng, The University of Hong Kong, Hong Kong

TITLE: Smale's Mean Value Conjecture and Related Problems

ABSTRACT: In this talk, we will explain how the theory of amoeba and geometric function theory can be used the study of Smale's mean value conjecture and other related problems.

Rikard Bøgvad, Stockholm University, Sweden

TITLE: Around Rodrigues' formula

ABSTRACT: Consider a probability measure μ with compact support in $\mathbb{C}$, sample the measure, producing n complex numbers $\xi_i, \imath = 1, 2, ..., n$ and a polynomial $S_n = \prod_{i=1}^{n}(z - \xi_i)$. Then differentiate this polynomial $[\alpha n]$ times producing

$$P_{[\alpha n],n}(z) = \frac{d^{[\alpha n]}}{dz^{[\alpha n]}} \left(S_n(z)\right),$$

where α is a positive number and $[\alpha n]$ the integer part of αn.

We study a (very) special case of the problem to describe what is the expected distribution of zeroes of $P_{[\alpha n],n}(z)$, namely the case when the measure μ is discrete, and the sampling deterministic. In that case the answer turns out to be surprisingly simple, in terms of an harmonic function of a rational plane curve, associated to the linear homogenous differential equation satisfied by $P_{[\alpha n],n}(z)$. This curve is also the algebraic equation satisfied by the Cauchy transform of the asymptotic root-counting measure for the sequence $\left\{P_{[\alpha n],n}\right\}$.

This is joint work with Christian Hägg and Boris Shapiro.

Christian Hägg, Stockholm University, Sweden

TITLE: Polynomials with roots at lattice points in a given plane polygon and their derivatives

ABSTRACT: Consider a complex polynomial P with simple zeros in lattice points contained in a simple polygon S. We numerically investigate how the zeros of $P; P', P'', \dots$ change, and notice that the order of derivation increases these zeros converge to some trees inside S.

Per Alexandersson, KTH, Sweden

TITLE: Around multivariate Schmidt-Spitzer theorem

ABSTRACT: Given an arbitrary complex-valued infinite matrix $A = (a_{ij})$, $i = 1, \dots, \infty; j = 1, \dots, \infty$ and a positive integer n, we introduce a naturally associated polynomial basis BA of $\mathbb{C}[x_0, \dots, x_n]$. We discuss some properties of the locus of common zeros of all polynomials in BA having a given degree m; the latter locus can be interpreted as the spectrum of the $m \times (m + n)$- submatrix of A formed by its m first rows and by the $(m + n)$ first columns. We initiate the study of the asymptotics of these spectra when $m \to \infty$ in the case when A is a banded Toeplitz matrix. In particular, we present and partially prove a conjectural multivariate analog of the well-known Schmidt-Spitzer theorem which describes the spectral asymptotics for the sequence of principal minors of an arbitrary banded Toeplitz

matrix. Finally, we discuss relations between polynomial bases BA and multivariate orthogonal polynomials.

Frantisek Stampach, FIT CTU in Prague, Czech Republic

TITLE: On the asymptotic zero distribution of orthogonal polynomials on the unit circle with variable Verblunsky coefficients

ABSTRACT: We study a sequence of OPUC whose Verblunsky coefficients depend on an additional parameter. We show that, in a certain limit, the asymptotic zero distribution of the studied polynomial sequence exists provided that the Verblunsky coefficients do not decay exponentially. The limiting measure is an average measure of the equilibrium measure of a circular arc. This is an analogous result to the known asymptotic distribution of OPRL with variable Jacobi parameters that was obtained earlier by A. Kuijlaars and W. Van Assche. We announce an open problem concerning the exponentially decaying Verblunsky coefficients. If time remains, we either discuss the second term of the asymptotic formula (strong Szegö limit theorem) or concrete examples.

Blagovest Sendov, Bulgarian Academy of Science, Bulgaria

TITLE: Sector Analogue of the Gauss-Lucas Theorem (joint with Hristo Sendov)

ABSTRACT: Recently we prove, see [9], the following:

Theorem 1. If the polynomial $p(z)$ is with real and non negative coefficients and has no zeros on the sector

$$S(\varphi) = \{z : \ z = re^{i\psi}, \ r > 0, \ |\psi| < \varphi, \ \varphi \in [0, \pi/2]\},$$

then $S(\varphi)$ do not contained any zero of the derivative $p'(z)$.

In the lecture we present a generalization of Theorem 1., by replacing the condition for non negativity of the coefficients of the polynomial.

We show that :

Theorem 2. If the coefficients of a monic polynomial $p(z)$ are in the sector

$$\{z : \ z = re^{i\psi}, \ r > 0, \ \psi \in [0, \phi], \ \phi \in [0, \pi)\},$$

and $p(z)$ has no zeros in the interior of this sector, then also the derivative $p'(z)$ has no zeros in the interior of that sector.

In addition, we give a necessary condition for a polynomial to satisfy the premise of Theorem 2.

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