Abstracts Non-Archimedean geometry, motives and vanishing cycles

Joseph Ayoub

On the conservativity conjecture

The conservativity conjecture predicts that an algebraic correspondence between Chow motives is invertible if and only if its action on cohomology (e.g., de Rham cohomology) is invertible. (This includes as particular cases the Bloch conjecture on surfaces and the Kimura-O'Sullivan finite dimensionality conjecture for smooth hypersurfaces in projective spaces.) I will report on work in progress aiming to prove the conservativity conjecture.

Vladimir Berkovich

Complex analytic vanishing cycles for formal schemes

Given a nonzero complex analytic function defined in a neighborhood of a point in a complex analytic space and equal to zero at the point, there are associated integral vanishing cycles cohomology groups. They are finitely generated abelian groups provided with a quasi-unipotent action of the fundamental group of the punctured complex plane. I will explain results which imply that those groups depend only on the formal completion of the local ring of the point. One can in fact extend the construction of those groups to arbitrary nonzero elements of the maximal ideal of the formal completion and, more generally, to a broad class of formal schemes over the formal completion of the ring of convergent power series at zero of the complex plane.

Alexis Bouthier

"Perverse sheaves on arc spaces"

In a joint work with D. Kazhdan, we study the geometry of arc spaces and its singularities, using a new tool pro-smooth morphisms.

In this context, we construct a six functor formalism and establish finitess results for the cohomology of these spaces, that are geometric incarnations of certain motivic integrals.

Antoine Ducros

Non-archimedean integrals as limits of complex integrals.

Every meromorphic one-parameter family X_t of complex projective varieties on the punctured unit disc gives rise to a t-adic analytic space in the sense of Berkovich, which is supposed to encode (part of) the limit behavior of the family (conjecture of Kontsevich and Soibelman, positive results by Berkovich, Nicaise, Boucksom and Jonsson...). In a joint work in progress with E. Hrushovski and F.

Loeser, we prove an instance of this general principle concerning integrals. More precisely, we start from an (n,n)-form ω in the sense of Chambert-Loir and myself on the "limit" t-adic analytic space (where n denotes its dimension), whose definition only involves invertible algebraic functions $f_1, \ldots,f_m$. We consider a compact polyhedral complex P in R^n , and set $V=(l\og |f_1|, \ldots,\log |f_n|)^{-1}(P)$. We define for every t a usual (n,n)-form on ω_t on X_t and a subset V_t of X_t, essentially by mimicking the definitions of ω and V, and by applying a re-normalization factor. We then prove than the integral of ω on V is equal to the limit at T goes to 0 of the classical integral of ω_t on V_t.

Tobias Dyckerhoff

Introduction to matrix factorizations

I will survey some aspects of the theory of matrix factorizations discussing their original appearance in Eisenbud's work and some more recent perspectives inspired by their role as B-branes in string theory.

Arthur Forey

Invariants of semi-algebraic sets in valued fields and virtual rigid motives

In a first part I will present Hrushovski and Kazhdan's theory of motivic integration. Then show how it is used by Hrushovski and Loeser to study invariants of the analytic Milnor fiber. In a second part I will present how one can use it to define a ring morphism from the Grothendieck group of semi-algebraic sets to the Grothendieck group of rigid analytic motives in the sense of Ayoub. This provides a notion of virtual rigid motive with compact support and can be related to Ayoub, Ivorra and Sebag's work on nearby motivic cycles.

Florian Ivorra

Motivic nearby sheaves, Milnor fibers and non-Archimedean geometry

J. Ayoub has shown that the theory of nearby cycles functors can also be developed in the motivic stable homotopy theory of schemes. In this talk, I will present joint works with J. Ayoub and J. Sebag relating motivic nearby sheaves with tubes in non-Archimedean geometry and virtual motivic Milnor fibers as introduced by J. Denef and F. Loeser using arc schemes and motivic integration. I will also explain a motivic interpretation of the weight zero part of the nearby sheaves.

Marc Levine

Lecture 1: *An introduction to motivic homotopy theory*

Lecture 2: *Computations in the motivic stable homotopy category*

Johannes Nicaise

A motivic Fubini theorem for the tropicalization map

Jérôme Poineau 1

Introduction to Berkovich analytic spaces

At the end of the eighties, Vladimir Berkovich introduced a new way to define p-adic analytic spaces. A surprising feature is that, although p-adic fields are totally discontinuous, the resulting spaces enjoy many nice topological properties: local compactness, local path-connectedness, etc. On the whole, those spaces are very similar to complex analytic spaces. They already have found numerous applications in several domains: arithmetic geometry, dynamics, motivic integration, etc.

We will review the basic theory of those spaces and cover in particular the following topics: affinoid algebras, affinoid spaces, description of the affine line, analytification of algebraic varieties.

Jérôme Poineau 2

Berkovich spaces over Z

Although Berkovich spaces usually appear in a non-archimedean setting, their general definition actually allows arbitrary Banach rings as base rings, e.g. Z endowed with the usual absolute value. Over the latter, Berkovich spaces look like fibrations that contain complex analytic spaces as well as p-adic analytic spaces for every prime number p. We will try to show what those spaces look like and explain their main properties both local (local path-connectedness, noetherianity of local rings, etc.) and global (vanishing of higher cohomology of disks, noetherianity of rings of global functions, etc.).

Mauro Porta

Representability theorem in derived analytic geometry

In this talk I introduce derived analytic spaces and their deformation theory. The goal is to explain how to formulate and prove a representability theorem in this context. The main motivation comes from enumerative geometry: in order to generalize the cylinder counting introduced by T. Y. Yu's talk to higher dimensional varieties, it is important to be able to construct a quasi-smooth derived structure on the moduli space of non-archimedean stable maps, which is the objective of an ongoing joint project with T. Y. Yu.

Marco Robalo

First talk:

Matrix Factorizations and derived algebraic geometry: a survey

In this talk we will follow the results of A. Preygel explaining how the 2-periodic structure on categories of matrix factorizations are a consequence of derived algebraic geometry.

Second Talk:

Motivic Realizations of categories of singularities and Vanishing Cycles

In this talk I will report a result obtained in collaboration with A. Blanc, B. Toen, and G. Vezzosi, establishing a link between the l-adic realization of the dg-categories of singularities and l-adic vanishing cycles.

Michael Temkin

"Covers of Berkovich curves and reduction of the different"

I will tell about a structure of wildly ramified covers

of Berkovich curves. This includes older results concerning

slopes of the different function, and new finer results about reduction

of differential forms and the different function, and their relation to

the dualizing sheaf and its reduction.

Alberto Vezzani

Realizations of rigid analytic motives and motivic Galois groups

We recall the construction of rigid-analytic motives over non-archimedean fields, and we define their de Rham and Betti realizations, in analogy with the archimedean situation. We also give an application of this formalism, extending a result of Ayoub on motivic Galois groups from the (equal) characteristic zero case to the positive and mixed characteristic case by means of Scholze's tilting equivalence.

Tony Yue Yu

The Frobenius structure conjecture in dimension two

The Frobenius structure conjecture is a conjecture about the geometry of rational curves in log Calabi-Yau varieties proposed by Gross-Hacking-Keel. It predicts that the enumeration of rational curves in a log Calabi-Yau variety gives rise naturally to a Frobenius algebra satisfying nice properties. In a joint work with S. Keel, we prove the conjecture in dimension two. Our method is based on the enumeration of non-archimedean holomorphic curves developed in my thesis. If time permits, I will also discuss the enumeration problem in higher dimensions, which is the motivation behind my joint works with M. Porta on derived non-archimedean analytic geometry.