Probabilistic Harmonic Analysis and Spectral Theory

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Abstracts

Christopher Thiele

Quantitative norm convergence of ergodic means for two commuting transformations

A transference principle allows to relate questions in ergodic theory with questions in harmonic analysis. Following this principle, convergence questions for bilinear ergodic averages with respect to two commuting transformations have lead to a particular program of study in harmonic analysis which has seen steady progress in recent years. We discuss several results including some joint work with P. Durcik, V. Kovac, and K. Skreb. on optimal quantitative norm convergence of ergodic averages for two commuting transformations.

Anton Baranov

Sampling, interpolation and Riesz bases in small Fock spaces

We give a complete description of Riesz bases of reproducing kernels in some Fock-type spaces. This characterization is in the spirit of the well-known Kadets--Ingham 1/4 theorem for Paley--Wiener spaces. Contrarily to the situation in Paley--Wiener spaces, a link can be established between Riesz bases in the Hilbert case and corresponding complete interpolating sequences in small Fock spaces with associated uniform norm. These results allow to show that if a sequence has a density stricly different from the critical one then either it can be completed or reduced to a complete interpolating sequence. In particular, this allows to give necessary and sufficient conditions for interpolation or sampling in terms of densities. The talk is based on a joint work with André Dumont, Andreas Hartmann and Karim Kellay.

Yuri Belov

Synthesable invariant subspaces

Let L be a proper differentiation invariant subspace of $C^\infty(a, b)$ such that restriction operator $\frac{d}{dx}|_L$ has a discrete spectrum Λ (counting with multiplicities).We characterize the subspaces satisfying spectral synthesis property (i.e. L is spanned by functions vanishing outside some closed interval $I\subset(a, b)$ and monomial exponentials corresponding to Λ). This holds if Beurling-Malliavin density of Λ does not exceed critical value or polynomials belong to some specific space of entire functions $H_{0,\Lambda}$ and are dense there. This result is sharp in some sense.
Amazingly, for the case when spectral synthesis fails all subspaces corresponding to fixed spectrum lambda are ordered by inclusion, and we are going to illustrate that. In addition, we will depict an intriguing connection between these problems and the theory of de Branges spaces, which, in its turn, is helpful in some other completeness problems.

Komla Domelevo

Differential subordination under change of law

We prove optimal L2 bounds for a pair of Hilbert space valued differentially subordinate martingales under a change of law.
The change of law is given by a process called a weight and sharpness in this context refers to the optimal growth with respect to the characteristic of the weight. The pair of martingales are adapted, uniformly integrable, and cadlag. Differential subordination is in the sense of Burkholder, defined through the use of the square bracket. In the scalar dyadic setting with underlying Lebesgue measure, this was proved by Wittwer in 1999, where homogeneity was heavily used. Recent progress by Thiele-Treil-Volberg and Lacey in 2015, independently, resloved the so-called non-homogenous case for predictable multiplier for discrete in time filtrations with two completely different proofs. The general case for continuous-in-time filtrations remained open and is adressed here. As a by-product, we give the explicit expression of a Bellman function of four variables for the weighted estimate of subordinate martingales with jumps.

Nicola Arcozzi

A probabilistic interpretation of some singular integrals in discrete environment

In this join work with Komla Domelevo and Stefanie Petermichl, we extend to some discrete settings the classical probabilistic interpretation of the Riesz transforms by Gundy and Varopoulos. Some L^p estimates follow.

Camil Muscalu

The helicoidal method

The plan of the talk is to describe a new method of proving (multiple) vector valued inequalities in harmonic analysis. It is conceptually simple and applicable to a wide range of problems. In particular, it allowed us to give positive answers to some natural open questions that have been circulating for some time. Joint work with Cristina Benea.

Cristina Benea

The Helicoidal Method II

We present recent joint with Camil Muscalu, and the talk should be regarded as a complement to the "Helicoidal Method". We will describe in more detail the method of the proof, as well as more recent developments.

Frederic Bernicot

Paracontrolled calculus and singular PDES through the heat semigroup

We will present the main idea of the paracontrolled calculus, which was
recently introduced in the Euclidean situation by Gubinelli, Imkeller and Perkowski.
This gives an alternative approach to Hairer's theory in order to deal with singular
(stochastic) PDEs, such as the Parabolic Anderson Model (2D-3D) and Burgers equations.
Indeed, it relies on the following fact: the paraproduct (as a singular bilinear operator)
is the well-suited analytic tool, in order to isolate the stochatic cancellations (which are then reduced
as a black box in the analysis of stochastic pdes) in nonlinearities.
We will then explain how we can extend it in many various situations, given by a heat semigroup with
gradient estimates. This work is joint with Ismael Bailleul and Dorothee Frey.

Francesco Di Plinio

Intrinsic positive sparse domination of multilinear singular integrals

In joint work with Amalia Culiuc (Brown/GATech) and Yumeng Ou (Brown/MIT), we develop
a novel approach to positive sparse domination of multilinear singular integrals based on
localized outer-L^p embedding theorems for wavelet and wave packet transforms. We refer
to our approach as "intrinsic" since, unlike the previous methods originating in the works
of Lerner and Lacey, it does not rely on any a-priori L^p or weak-L^p estimates, and produces
a sparse form which only depends on the input functions, and can be explicitly described. The prime
application of our principle is to modulation invariant operators, such as the bilinear Hilbert
transforms and the variational Carleson operator (joint with Gennady Uraltsev, U Bonn), which are
not treatable by the standard methods. In particular, we obtain as a corollary a completely novel
and rich multilinear weighted theory for these classes of modulation invariant operators. When
applied in the context of multilinear Calderon-Zygmund operator, our methods yield a T(1) type
theorem which strengthens the results of Lerner-Nazarov and Conde-Rey to a domination of uniform
(i.e. independent of the CZ kernel within the same smoothness class) type.

Eero Saksman

On the Riemann zeta function and multiplicative chaos

We describe our current understanding of the connection between statistics of the Riemann zeta
function over the critical line and Gaussian chaos measures. The talk is based on joint work with
Christian Webb (Aalto University).

Sergey Treil

Bellman function and the Carleson Embedding Theorem with matrix weights

The Carleson Embedding Theorem, and especially its dyadic (martingale) versions are cornerstone of
the modern harmonic analysis. For the sharp weighted estimates with matrix weights of singular
integral operators one needs a version of the Carleson Embedding theorem with matrix weights both
in the domain and in the target space. In the case when only one of the weights is matrix-valued, the
result can be trivially obtained from the scalar Carleson Embedding Theorem. But the case when
both weights are matrix-valued is much trickier, and requires more effort.

I will present a proof of this result, based on he Bellman function technique; it is interesting than no other known method of the proof for the scalar case does not appear to work in the matrix case. The talk is based on a joint work with A. Culiuc.

Joaquim Ortega

Determinantal point processes in the torus and the sphere

We study expected Riesz s-energies and linear statistics of some determinantal processes on the sphere and the torus. In particular, we compute the expected Riesz and logarithmic energies of the determinantal processes given by the reproducing kernel of the space of spherical harmonics in the sphere. This kernel defines the so called harmonic ensemble on the sphere. With these computations we improve previous estimates for the discrete minimal energy of configurations of points in the sphere. We prove a comparison result for Riesz 2-energies of points defined through determinantal point processes associated to isotropic kernels. As a corollary we get that the Riesz 2-energy of the harmonic ensemble is optimal among ensembles defined by isotropic kernels with the same trace. Finally, we study the variance of smooth and rough linear statistics for the harmonic ensemble and compare the results with the variance for the spherical ensemble and the determinantal processes given by Bochner-Riesz kernels.

Alan Sola

Disks and slits: scaling limits in conformal aggregation models

I will report on work in progress with A. Turner (Lancaster) and F. Viklund (KTH) on a natural variation of the Hastings-Levitov model of random aggregation in the plane.
I will explain how, for certain parameter values and under a sufficiently weak regularization, clusters obtained by composing certain random conformal maps can be shown to converge to a randomly oriented one-dimensional slit in a small-particle limit. This contrasts with earlier work in which scaling limits, under sufficiently strong regularizations, have been shown to be deterministically growing disks.

Aron Wennman

Discrepancy densities for geometric zero packing

I will discuss the problem of geometric zero packing, recently introduced by Hedenmalm in connection with work on the universal asymptotic variance.
Loosely, the problem is to approximate certain functions, such as the hyperbolic density $1/(1-|z|^2)$, with the modulus of polynomials. The problem can also be phrased in terms of optimally discretizing a smooth positive density as a sum of point masses (corresponding to the zeros of the polynomial)
In a recent preprint we study certain discrepancy densities associated to this problem, which measure the failure in such an approximation. Two such densities are of particular importance: the so-called hyperbolic- and tight hyperbolic discrepancy densities.
We prove that these are equal, which gives some insight into the behaviour of optimal polynomials.

Evgueni Abakoumov

Proper holomorphic embeddings, radial approximation, and tropical power series

We study the problem of approximation of radial weights by finite sums of moduli of holomorphic functions. Relations to other approximation problems and to tropical power series are discussed.
As an application, quantitative aspects of proper holomorphic embeddings of planar domains and complex balls into CN are considered. The talk is based on joint work with E. Dubtsov.

Emmanuel Fricain

Multipliers between model spaces

Motivated by a question from operator theory, we discuss the problem of multipliers from one model space to another. We give a characterization involving kernels of Toeplitz operators and Carleson measures. We also discuss a question of Cohn concerning the boundedness of these multipliers which are (contrary to the case when one multiplies a reproducing kernel Hilbert space to itself) not automatic. This is a joint work with Andreas Hartmann and Bill Ross.

Kelly Bickel

Two Weight Estimates for Well Localized Operators

In this talk, we will discuss two-weight estimates for well localized operators in the setting of both A2 weights and more general matrix measures and atomic filtrations. The main result says that Sawyer-type testing conditions (i.e. the boundedness of the operator and its adjoint on characteristic functions of cubes) are necessary and sufficient for the boundedness of such operators, including examples like Haar Shifts and paraproducts.
This is joint work with Amalia Culiuc, Sergei Treil, and Brett Wick.