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Poster session

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List of posters

Abstracts

Surface family with a common natural geodesic lift

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In the present paper, we find a surface family possessing the natural lift of a given curve as a geodesic. We express necessary and sufficient conditions for the given curve such that its natural lift is a geodesic on any member of the surface family. We present a sufficient condition for ruled surfaces with the above property. Finally, we illustrate the method with some examples.

This is joint work with Evren Ergün.

Cohomology of Arrangements, Moduli Spaces of Curves and Moduli Spaces of Del Pezzo Surfaces

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An arrangement is a finite set of closed subvarieties of a larger variety. Despite their simple definition, arrangements are of interest to number of areas of mathematics such as algebraic geometry, combinatorics, Lie theory, singularity theory and topology and many interesting spaces can be described in terms of arrangements.
Classically, most attention has been given to arrangements of hyperplanes in an affine or projective space but the past two decades toric arrangements, i.e. arrangements of codimension 1 subtori in a larger torus, have gained interest. Here, we compute the cohomology of the complement of toric arrangements and use these results to compute cohomology of certain moduli spaces of curves and Del Pezzo surfaces.

Infinitary intuitionistic logics

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Systems for classical infinitary propositional and first-order logic have been described and studied extensively by Carol Karp (1964). Intuitionistic versions of infinitary logics using countable many conjunctions and disjunctions were studied by Mark Nadel, among others (1978), and the completeness with respect to the infinitary Kripke semantics was established. Our purpose is to study the general case of infinitary logics without the axiom of excluded middle and to get analogues of the completeness results.

Dynamics of Derivations

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Linear dynamics has been a rapidly evolving area of operator theory since the late 1980s.
The primary goal is to understand the hypercyclic properties of commutator maps, or inner derivations, $S \mapsto AS - SA$ and generalised derivations $S \mapsto AS - SB$, for fixed bounded linear operators A and B, on ideals of operators. Hitherto in this setting the only results have been the characterisation of the hypercyclicity of the left and right multipliers.
We present concrete families of hypercyclic generalised derivations and the main non-trivial example will surprisingly reveal that scalar multiples of the backward shift operator, which satisfy the Hypercyclicity Criterion on the sequence space ℓ^2, do not induce a hypercyclic commutator map on the space of compact operators on ℓ^2.

This is joint work with Eero Saksman and Hans-Olav Tylli.

Fractional Derivative of Riemann $\zeta-$Function and Main Properties

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Caputo-Ortigueira fractional derivative provides the fractional derivative of complex functions. It is defined in [3, 4] as

$$_c\mathrm{D}^\alpha f(s) = {_o\mathrm{D}^{\alpha-m}}\{f^{(m)}(s)\} = \frac{e^{i(\pi-\theta)(\alpha-m)}}{\Gamma(m-\alpha)} \int_0^\infty \frac{d^m}{ds^m} \frac{f(xe^{i\theta}+s)}{x^{\alpha-m+1}} dx$$

where $s \in \mathbb{C}$, f is a complex function and $m-1 < \alpha < m \in \mathbb{Z}^+$, $\theta \in [0, 2\pi)$ and $_o\mathrm{D}^\alpha$ is the Ortigueira derivative operator [4]. This derivative plays an important role in the number theory, and has been shown suitable for the analysis of the Dirichlet series, Hurwitz zeta function $\zeta(s,a)$ [1] and Riemann zeta function $\zeta(s)$ [6]: the fractional derivatives of these two last functions are [1, 2]

$$\zeta^{(\alpha)}(s,a) =_c \mathrm{D}^\alpha\{\zeta(s,a)\} = e^{i\pi\alpha} \sum_{n=0}^{\infty} \frac{(\log(n+a))^\alpha}{(n+a)^s}$$

$$\zeta^{(\alpha)}(s) =_c \mathrm{D}^\alpha\{\zeta(s)\} = e^{i\pi\alpha} \sum_{n=1}^{\infty} \frac{(\log n)^\alpha}{n^s}$$

in which $0 < a \leq 1$. The convergence of the Riemann zeta function is studied in [2]. In particular, this fractional derivative is equal to a complex series, whose real and imaginary parts are convergent in a half-plane which depends on the order of the fractional derivative. An integral representation for $\zeta^{(\alpha)}$ was discovered. Since the Riemann zeta function is widely used in Physics [5, 7, 8, 9], the unilateral Fourier transform of $\zeta^{(\alpha)}$ is computed to investigate its applications in Quantum Theory and Signal Processing.

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Mathematical Formulae Beyond the Tree Paradigm

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A restriction in the traditional language of mathematics is that every formula has an underlying tree structure, wherein an expression decomposes into subexpressions that each contribute one result to the whole. Some modern mathematics (especially in higher algebra and differential geometry) is however not so well catered for by this tree paradigm, and would be much better off if this restriction was lifted; it is not that the mathematics *requires* a generalised formula language, but it becomes a whole lot easier to state and do when you use one.
This poster shows the formula language of *networks* that arises when one relaxes the tree restriction to instead allow an underlying directed acyclic graph (DAG) structure. Evaluation of networks is non-obvious, but can be carried out in any PROP/symocat; conversely, evaluation of networks characterises PROPs in much the same way as evaluation of words characterises monoids. Networks are isomorphic to the abstract index interpretation of Einstein convention tensor expressions, and demonstrate that these can in fact be given a coordinate-free interpretation. Special cases of the network notation that have been invented independently include the Penrose graphical notation and quantum gate arrays.

$K-$structure of the $U(\mathfrak{g})^K-$module $U(\mathfrak{g})$ for simple Lie algebras $\mathfrak{g} = \mathfrak{su}(n,1)$ and $\mathfrak{g} = \mathfrak{so}(n,1)$

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Let $\mathfrak{g}$ be a simple real Lie algebra, $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ its Cartan decomposition, G the adjoint group of $\mathfrak{g}$, K its maximal compact subgroup with Lie algebra $\mathfrak{k}$. Further, denote by $U(\mathfrak{g})$ and $U(\mathfrak{k}) \subseteq U(\mathfrak{g})$ the complexified universal enveloping algebras of $\mathfrak{g}$ and $\mathfrak{k}$ and let $Z(\mathfrak{g})$ and $Z(\mathfrak{k})$ be its centers. Let $U(\mathfrak{g})^K$ be the subalgebra of $K-$invariants in $U(\mathfrak{g})$. Then obviously we have a morphism of algebras $Z(\mathfrak{g}) \otimes Z(\mathfrak{k}) \longrightarrow U(\mathfrak{g})^K$ defined by the multiplication. Knop has proved that for noncompact $\mathfrak{g}$ this morphism is always injective and that its image is exactly the center of the algebra $U(\mathfrak{g})^K$. Furthermore, the algebra $U(\mathfrak{g})^K$ is commutative, i.e. isomorphic to $Z(\mathfrak{g}) \otimes Z(\mathfrak{k})$, if and only if $\mathfrak{g}$ is either $\mathfrak{su}(n,1)$ or $\mathfrak{so}(n,1)$. In these cases $U(\mathfrak{g})$ is free as a $U(\mathfrak{g})^K-$module. We show that in these cases the multiplication defines an isomorphism of $K-$modules and $U(\mathfrak{g})^K-$modules $U(\mathfrak{g})^K \otimes H \longrightarrow U(\mathfrak{g})$, where H is the subspace of $U(\mathfrak{g})$ spanned by all powers x^k, $k \in \mathbb{Z}_+$, $x \in \mathcal{N}_K$, and $\mathcal{N}_K$ is the variety of all nilpotent elements in $\mathfrak{g}^{\mathbb{C}}$ whose projection to $\mathfrak{k}^{\mathbb{C}}$ along $\mathfrak{p}^{\mathbb{C}}$ is nilpotent in the reductive Lie algebra $\mathfrak{k}^{\mathbb{C}}$. Furthermore, we study the structure of the $K-$module H and show that the multiplicity of every irreducible representation δ of K in it equals its dimension $d(\delta)$. In other words, as a $K-$module H is equivalent to the regular representation of K.

The foundation of delayed population dynamics: monotone dynamics or classical problems?

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The logistic equation is often considered as a simplification of the chemostat (Kooi, Boer, Kooijman (1998)). However, the global qualitative properties of the delayed single species chemostat are completely known, see e. g. Smith (2011). Such properties still remain open for the delayed logistic equation, see e. g. Bánheley, Czendes, Kristin, and Neymaier (2014) despite that they have been announced a long time ago (Wright (1955)). We may therefore ask whether the logistic equation really is a simplification and what information about the chemostat actually is contained. We discuss the links between these equations and conclude that they are less clear in the delayed case than they are in the non-delayed case. Our main conclusion is that monotone dynamics apply in many cases that can be mechanistically motivated in order to elucidate the global properties of the solutions.

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A Formal Language of Type Theory of Situated Information

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We introduce a formal language L^{ST} of typed, situated information. The L^{ST}-terms represent situation-theoretic information about relations taking (or not taking) place between objects, in space-time locations, in small, partial situations. (1) The language L^{ST} includes terms of generalized recursion over relational structures. The *recursion scope* of a *recursion term* designates instantiations of recursion variables by assigning information to them, via mutual recursion. The recursion assignments are mathematical representation of saving information in memory cells. (2) L^{ST} has terms with components that designate propositional restrictions over objects. (3) In addition, L^{ST} has generalized, restricted variables, as formal representation of *memory networks*. These complex objects are formalized by restricted, recursion terms, the head part of which is a set of recursion variables restricted to simultaneously satisfy constraints, along with recursion assignments.
L^{ST} formalizes the concepts of information, which is obtained algorithmically, by recursion. Information is subject of restrictions and saved in memory cells. When the restrictions are not satisfied, the algorithm for collecting and saving information is undefined or terminates with an error flag. Memory cells can form restricted memory networks.

Reduction Calculi in Type Theory of Acyclic Recursion and Applications

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In 1994, Yiannis Moschovakis initiated development of a new approach to the mathematical notion of algorithm, starting with the untyped version. In 2006, he introduced the simply-typed theory of acyclic recursion, L^{λ}_{ar}. The formal language of L^{λ}_{ar} has two kinds of semantics — denotational and algorithmic. The reduction calculus of L^{λ}_{ar} is instrumental in algorithmic semantics, by providing effective reduction of each L^{λ}_{ar}-term to its canonical form. The canonical form $\mathsf{cf}(A)$ of a meaningful L^{λ}_{ar}-term A determines the abstract, mathematical *algorithm* for computing its semantic *denotation* $\mathsf{den}(A)$.
In our work, we extend the reduction calculus of L^{λ}_{ar} and its referential equivalence to γ-reduction calculus and γ-equivalence, respectively. The extended γ-reduction has applications to semantic analysis of formal and natural languages. We present analysis of typical expressions of human language, via syntax-semantics interface in computational grammar.

Rigidity of Extremal Quasiregularly Elliptic Manifolds

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e show that for a closed n-manifold N admitting a quasiregular mapping from Euclidean n-space the following are equivalent: (1) order of growth of $\pi_1(N)$ is n, (2) N is aspherical, and (3) $\pi_1(N)$ is virtually $\mathbb{Z}^n$ and torsion free.

This is joint work with Pekka Pankka.

What is the distance of a Volterra-type integral operator from compact operators?

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We consider a Volterra-type integral operator

$$T_g f(z) = \int_0^z f(\zeta) g'\zeta d\zeta, \quad z \in \mathbb{D},$$

defined on a large class of weighted Bergman spaces. The operator T_g was introduced by Ch. Pommerenke and it has been studied systematically by several people including A. Aleman, A.G. Siskakis and R. Zhao among others. A natural problem is to estimate the essential norm of T_g, the distance from compact operators in the operator norm, on different spaces of analytic functions. Quantitative estimates for the essential norm of T_g on standard Bergman spaces A^p_α have been established by J. Rättyä in 2007. We discuss an extension of these results to a larger class of weighted Bergman spaces A^p_ω where the weight ω satisfies a certain doubling property.
This poster is based on joint work with Pekka Nieminen from the University of Helsinki and Wen Xu from the University of Eastern Finland.

Generalization of the Ramanujan's Pythagorean identities

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In Notebook II, Chapter XXI, Srinivasa Ramanujan establish the identities $2(\sqrt{a^2+b^2}-a)(\sqrt{a^2+b^2}-b)=(a+b-\sqrt{a^2+b^2})^2$ and $3(\sqrt[3]{a^3+b^3}-a)(\sqrt[3]{a^3+b^3}-b)=(\sqrt[3]{(a+b)^2}-\sqrt[3]{a^2-ab+b^2})^3$, with respect to a right triangle of sides a, b and hypotenuse $\sqrt[2]{a^2+b^2}$. Here, we prove that the identity $\sum_{k=1}^{n}(-1)^{k+1}\frac{n!}{k!(n-k)!}((\sqrt[n]{a^n+b^n})^{n-k}-a^{n-k})(\sqrt[n]{a^n+b^n}-b)^k=(a+b-\sqrt[n]{a^n+b^n})^n$ is a natural generalization of previous identities and that the use of Antiphon-Archimedes' principle establishes a logical equivalence between these identities and the Pythagorean theorem. New elementary type of construction of Pythagorean triples is established.

This is joint work with Liuhan de Oliveira Miranda and Lossian Barbosa Bacelar Miranda.

Some Formally Self-Dual Codes over a Ring of Characteristic 2 and Their Construction Methods

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In this work, Gray images of formally self-dual codes over the ring $\mathcal{S}_4 = \mathbb{F}_2 + u\mathbb{F}_2 + u^2\mathbb{F}_2 + u^3\mathbb{F}_2 \simeq \mathbb{F}_2[u]/\langle u^4 \rangle$ and some of their construction methods are considered. Binary codes with large automorphism groups are obtained as Gray images of extremal formally self-dual codes over $\mathcal{S}_4$ by using some computer algorithms in MAGMA computer program, which is a large, well-supported software package designed for computations in algebra, coding theory, number theory, algebraic geometry and algebraic combinatorics.

This is joint work with Bahattin Yıldız.

On the real rank of monomials

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Due to several applications in different fields of pure and applied mathematics, we study Waring decompositions of polynomials, namely decompositions as sums of powers of linear forms. In this poster, we present a recent work where we compare Waring decompositions of monomials over the real and the complex numbers.
This is a joint work with E.Carlini, M.Kummer and E.Ventura.

On certain topological spaces and related geometric visualizations

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This study focuses on certain topological and geometric properties of certain linear topological spaces that constitute a very general class of metrizable locally convex linear spaces. Visualization of the geometric structures of the spaces is achieved by the graphical representation of neighbourhoods.

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The spectra of linear fractional composition operators on weighted Dirichlet spaces

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We compute the spectra of composition operators C_ϕ, $f \mapsto f \circ \phi$, acting on certain scales of weighted Dirichlet spaces. Here the inducing map ϕ is a hyperbolic or parabolic automorphism or a parabolic non-automorphism of the unit disc. Our results answer to the remaining cases in this context.

This is joint work with Eva Gallardo-Gutierréz.

Hausdorff dimension of sets of generalized continued fractions with bounded partial coefficients

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Let us denote

$$E_{\mathcal{B}}^{\mathcal{A}} = \left\{ \underset{n=1}{\overset{\infty}{\mathrm{K}}} \frac{a_n}{b_n} \mid a_n \in \mathcal{A},\, b_n \in \mathcal{B} \right\},$$

where $\mathcal{A}$ and $\mathcal{B}$ are non-empty, finite sets of positive integers. We show that when $\mathcal{A}$ and $\mathcal{B}$ satisfy a simple criterion, the Hausdorff dimension of the set $E_{\mathcal{B}}^{\mathcal{A}}$ can be calculated with high precision using the periodic point method of O. Jenkinson and M. Pollicott. The dimensions of several example sets are calculated with rigorous error bounds.

This is joint work with Jaroslav Hančl, Kalle Leppälä and Tapani Matala-aho.

Upper estimate of the resolvent of a class of nonselfadjoint rational operator functions

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Let $A, B : \mathcal{H} \to \mathcal{H}$ be selfadjoint operators in a Hilbert space $\mathcal{H}$ and assume that B is bounded. Define the rational operator function

$$\mathcal{S}(\omega) = A - \omega^2 - \frac{\omega^2}{c - id\omega - \omega^2} B, \quad \text{dom } \mathcal{S}(\omega) = \text{dom A}, \quad \omega \in \mathbb{C} \setminus \text{P},$$

where $c \geq 0$, $d > 0$ and P is the set of poles of the rational function. Inspired by the definition of pseudospectra, we introduce a pseudonumerical range and study an enclosure of this set. The enclosure of the pseudonumerical range provides an upper bound on the norm of the resolvent of $\mathcal{S}$. This bound is computed exactly and is optimal for known numerical ranges of A and B.