Adaptive evolution and concentrations in parabolic PDEs

Benoît Perthame
Université Pierre et Marie Curie, Paris, France

Living systems are characterized by variability; in the view of C. Darwin, they are subject to constant evolution through the three processes of population growth, selection by nutrients limitation and mutations.

Several mathematical theories have been proposed in order to describe the dynamics generated by the interaction between their environment and the trait selection of the 'fittest'. One can use stochastic individual based models, dynamical systems, game theory considering traits as strategies. From a populational point of view, the population obeys an integro-partial-differential equation for the density number.

We will give a self-contained mathematical model of such dynamics and show that an asymptotic method allows us to formalize precisely the concepts of monomorphic or polymorphic population. Then, we can describe the evolution of the 'fittest trait' and eventually to compute various forms of branching points which represent the cohabitation of two different populations.

The concepts are based on the asymptotic analysis of the above mentioned parabolic equations once appropriately rescaled. This leads to concentrations of the solutions and the difficulty is to evaluate the weight and position of the moving Dirac masses that describe the population. We will show that a new type of Hamilton–Jacobi equation, with constraints, naturally describes this asymptotic.

Recent developments concern non-proliferative advantages and lead to define the notion of 'effective fitness'.