

Discrete Mathematics & Combinatorics

Warmth and Hom complexes of graphs

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In this talk, we study different general approaches for finding lower bounds on the chromatic number of graphs. The study takes off from the neighbourhood complexes introduced by Lovász in his proof of the Kneser conjecture, and their modern relatives. We compare this approach to a novel one, introduced by Brightwell and Winkler, who study notions of ‘long range action’ of graph homomorphisms, motivated by constructions in statistical physics. They introduce a parameter called the warmth, which constitutes a lower bound of the chromatic number.

We show that the lower bounds obtained by the two seemingly different approaches share several common features. In particular, we show that if the neighbourhood complex has an infinite first homology group, then the warmth is at most three. This is a first non-trivial case of the conjecture by Kahle that the warmth of a graph G is always less than three plus the connectivity of neighbourhood complex. We also provide further evidence for this conjecture, giving upper bounds for warmth, analogous to known bounds for connectivity of neighbourhood complexes.

This is joint work with Anton Dochtermann.