Dynamical Systems

Stability and perturbations of countable Markov maps

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Given an expanding countable Markov map (for example the Gauss map, Lüroth map, induced map for the intermittent Manneville–Pomeau system) we study the topological stability of the map when a small pointwise perturbation is applied. In particular, we study the singularity of the topological conjugacy between the original countable Markov map and the perturbed map. Often the topological conjugacy is a singular function (derivative is zero a.e.) and for many examples their properties are well-studied, most famously for the Minkowskis question mark function, which is a topological conjugacy between the Gauss map and a Lüroth map.
Due to the non-compact nature of countable Markov maps, many standard notions of singularity of the conjugacy behave discontinuously under perturbations. This happens for example for the Hlder exponent of the conjugacy or Hausdorff dimension of the pullback of the absolutely continuous invariant measure under the conjugacy, which can attain any possible values no matter how small the perturbation is. However, we found out that the Hausdorff dimension of the set of points where the derivative of the conjugacy is not zero instead does behave continuously under pointwise perturbations given suitable regularity assumptions on the maps. Using this result we also established a continuity theorem in non-uniformly hyperbolic dynamics for topological conjugacies between Manneville–Pomeau maps under perturbations.

This is joint work with Thomas Jordan (Bristol) and Sara Munday (Bologna).