INSTITUT
MITTAG-LEFFLER
THE ROYAL SWEDISH ACADEMY OF SCIENCES

Eigenvalues and Inequalities

14 - 18 May 2018

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2 Abstracts

Cwikels's bound reloaded

Dirk Hundertmark
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There are a couple of proofs by now for the famous Cwikel Lieb Rozenblum (CLR) bound, which is a semiclassical bound on the number of bound states for a Schrödinger operator, proven in the 1970s.

Of the rather distinct proofs by Cwikel, Lieb, and Rozenblum, the one by Lieb yields the best constant, the one by Rozenblum does not seem to yield any good or even reasonable estimate for the constants, and Cwikel's proof, which in a sense is the most elementary, is said to yield a constant which is at least about 2 orders of magnitude off the truth. This situation did not change much during the last 40+ years, even though there were other proofs of the CLR inequality later, the most radical one based on ideas of Rumin in the work by Rupert Frank.

We show that this common belief, i.e, Cwikel's approach yields bad constants, is not set in stone, by giving a drastic simplification of Cwikel's original approach which leads to a rather good bound for the constant in the CLR inequality. Moreover, our proof highlights the connection of the CLR bound with maximal Fourier multiplier estimates from harmonic analysis.

This is joint work with Peer Kunstmann, Tobias Ried, and Semjon Wugalter.

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On the semi-classical analysis of the groundstate energy of the Dirichlet Pauli operator (after Helffer-P. Sundqvist and Helffer-Kovařík-P. Sundqvist)

Bernard Helffer
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Motivated by a recent paper by Ekholm-Kovařík-Portmann, we analyze the semi-classical analysis of the ground state energy of the Dirichlet-Pauli operator. Tunneling effect can be measured with some analogy with the semi-classical analysis of the small eigenvalues of a Witten Laplacian, as analyzed in papers by Helffer-Sjöstrand, Helffer-Klein-Nier, Helffer-Nier,....

The presented works are in collaboration with Mikael Persson Sundqvist (University of Lund) and Hynek Kovařík.

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Interpolation inequalities and spectral Estimates for Magnetic Operators. Magnetic Rings.

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To be announced.

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The Excess Charge of Atoms

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We bound the excess charge of atoms described by the Vlassov equation and the time-dependent Thomas-Fermi equation. The bound states are defined in a dynamic way. The proof is inspired by an unpublished idea of Benguira in the time-independent setting which has earlier been used by Lewin and Lenzmann in the linear time-dependent setting. The talk is based on joined work with Li Chen (Mannheim).

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Spectrum of Dirac operators describing Graphene Quantum dots

Hanne Van Den Bosch
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Low energy electronic excitations in graphene, a two-dimensional lattice of carbon atoms, are described effectively by a two-dimensional Dirac operator. For a bounded flake of graphene (a quantum dot), the choice of boundary conditions determines various properties of the spectrum. Several of these choices appear in the physics literature on graphene. For a simply connected flake and a family of boundary conditions, we obtain an explicit lower bound on the spectral gap around zero. We also study the effect of the boundary conditions on eigenvalue sums in a semi-classical limit in the form of Weyl asymptotics. This is joint work with Rafael Benguria, Søren Fournais and Edgardo Stockmeyer.

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Limiting absorption principle and spectral asymptotics for waveguides with singular perturbations

André Froehly
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We consider a general second-order elliptic differential operator on a domain with a cylindrical end. We impose Dirichlet conditions on the boundary with the exception of a small set, where we impose Neumann boundary conditions. We prove a corresponding limiting absorption principle for the resolvent and show an asymptotic formula for the resonances in the case that the Neumann window shrinks to a point.

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Regularity of the optimal sets for spectral functionals

Susanna Terracini
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In this talk we deal with the regularity of optimal sets for a shape optimization problem involving a combination of eigenvalues, under a fixed volume constraints. As a model problem, consider

$$\min\Big\{\lambda_1(\Omega)+\cdots+\lambda_k(\Omega)\ :\ \Omega\subset\mathbb{R}^d,\ \text{open}\ ,\ |\Omega|=1\Big\},$$

where $\lambda_i(\cdot)$ denotes the eigenvalues of the Dirichlet Laplacian and $|\cdot|$ the d-dimensional Lebesgue measure. We prove that any minimizer Ω_{opt} has a regular part of the topological boundary which is relatively open and $C^{1,\alpha}$ regular and that the singular part has Hausdorff dimension smaller than $d-d^*$, where $d^*\geq 3$ is the minimal dimension allowing the existence of minimal conic solutions to the bow-up problem. We shall discuss similar problems for more general spectral functionals $F(\lambda_1(\Omega),\dots,\lambda_k(\Omega))$.

We mainly use techniques from the theory of free boundary problems, which have to be properly extended to the case of vector-valued functions: nondegeneracy property, Weiss-like monotonicity formulas with area term; finally through the properties of non tangentially accessible domains we shall be in a position to exploit the "viscosity" approach recently proposed by De Silva.

This is a joint work with Dario Mazzoleni and Bozhidar Velichkov.

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A Faber-Krahn inequality for the Robin Laplacian on exterior domains

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We will discuss generalizations of the Faber-Krahn inequality for the lowest eigenvalue of the Robin Laplacian with a negative boundary parameter on the exterior $\mathbb{R}^d \setminus \overline{\Omega}$ to a bounded, simply connected, smooth domain $\Omega \subset \mathbb{R}^d$. Our main motivation is to go beyond more traditional bounded domains in eigenvalue optimization.

The ultimate goal is to prove that the exterior of a ball maximizes the underlying lowest eigenvalue under a suitable constraint being imposed. In two dimensions, we constrain either the perimeter of Ω or its area. Constraining either the area of $\partial\Omega$ or the volume of Ω lead in higher dimensions to ill-posed optimization problems. Instead, we constrain for $d \geq 3$ the ratio between a Willmore-type energy of $\partial\Omega$ and its area, in addition, assuming convexity of Ω.

In the proof, we express the lowest eigenvalue via the min-max principle on the level of quadratic forms, rewritten in parallel coordinates on $\mathbb{R}^d \setminus \overline{\Omega}$ or in their modifications. The trickiest part of the proof is to find a proper test function.

In two dimensions, we will discuss two more geometric settings. First, we will consider the case of disconnected Ω's, in which the number of connected components enters the geometric constraint. Secondly, we will consider the case of Ω being an open arc, whose exterior $\mathbb{R}^2 \setminus \overline{\Omega}$ can be viewed as a plane with a cut.

These results are obtained in [L] and in [KL1, KL2], jointly with David Krejčiřík.

[KL1] D. Krejčiřík and V. Lotoreichik, Optimisation of the lowest Robin eigenvalue in the exterior of a compact set, *J. Convex Anal.* **25** (2018), 319–337.

[KL2] D. Krejčiřik and V. Lotoreichik, Optimisation of the lowest Robin eigenvalue in the exterior of a compact set, II: non-convex domains and higher dimensions, *submitted*, arXiv:1707.02269.

[L] V. Lotoreichik, Spectral isoperimetric inequalities for singular interactions on open arcs, *to appear in Appl. Anal.*, arXiv:1609.07598.

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Eigenvalues of a Robin Laplacian with a large parameter in the boundary condition: recent results for non-smooth domains

Konstantin Pankrashkin
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Let U be an bounded domain. We will be interested in the asymptotics of the Laplacian eigenvalues in U for the boundary condition $D_n u = au$, where D_n is the outward normal derivative and $a > 0$ is a large parameter. The problem was studied intensively during the last 5 years by various authors, including several participants of the present meeting, and as a starting point we give a short review of results on the role of the boundary regularity and on asymptotic versions of Faber-Krahn-type inequalities. We discuss then detailed eigenvalue asymptotics for 2D domains with corners and, if time permits, for domains with

cusps, as obtained in the most recent works jointly with Hynek Kovarik, Thomas Ourmieres-Bonafos and Magda Khalile.

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Maximization of Neumann eigenvalues

Dorin Bucur
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In this talk I will discuss the question of the maximization of the k-th eigenvalue of the Neumann-Laplacian among open sets with fixed volume. After an introduction to the topic and discussion about the existence of optimal geometries, I will focus on the low eigenvalues. The first non-trivial one is maximized by the ball, the result being due to Szego and Weinberger in the fifties.

Concerning the second non-trivial eigenvalue, Girouard, Nadirashvili and Polterovich proved that the supremum in the family of planar simply connected domains of R^2 is attained by the union of two disjoint, equal discs. I will show that a similar statement holds in any dimension and without topological restrictions. In particular, this implies that the Polya conjecture for the Neumann eigenvalues holds for the second eigenvalue and for arbitrary domains.

This last result is jointly obtained with A. Henrot.

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Maximizing Riesz means of Dirichlet eigenvalues

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During recent years the problem of minimizing λ_k the k-th eigenvalue of the Dirichlet Laplacian among open sets of fixed measure has seen much progress. However, a precise description of the minimizers is still far out of reach. Numerical evidence suggests that the minimizers need not even have any clear symmetries. It is thus natural to ask whether some structure emerges as k becomes large. In this talk we shall discuss a weaker version of this question. Specifically, we shall consider the behaviour, as $\lambda \to \infty$, of domains maximizing

$$\mathrm{Tr}(-\Delta_\Omega - \lambda)_-^\gamma = \sum_{\lambda_k < \lambda} (\lambda - \lambda_k(\Omega))^\gamma,$$

for $\gamma \geq 1$ among convex domains of fixed measure.

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Asymptotically optimal eigenvalues of the Robin Laplacian

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A recent result of Colbois and El Soufi shows that to prove Pólya's conjecture that the kth eigenvalue of the Dirichlet (Neumann) Laplacian on any domain always lies above (below) the corresponding first term in the Weyl asymptotics, it is sufficient to show that among all domains of given volume the ball asymptotically minimises (maximises) the kth eigenvalue as $k \to \infty$ in a certain sense. While we seem to be a long way from having the tools necessary to prove this, it has sparked considerable interest in determining spectrally asymptotically optimal domains within special classes such as rectangles or triangles, beginning with a paper of Antunes and Freitas in 2013. This, in turn, has opened up new links to lattice point counting problems.

After giving a brief overview of these results and connections, we will present analogues for the Laplacian with Robin boundary conditions. Here the picture is actually completely different: there is no direct equivalent of Pólya's conjecture; instead, the disjoint union of k equal balls is conjectured to minimise the kth eigenvalue for k sufficiently large.

We show that among rectangles and unions of rectangles the disjoint union of k equal squares often minimises the kth eigenvalue, and explore a number of consequences of this, including for the form of the general conjecture. This is based on joint work with Pedro Antunes and Pedro Freitas.

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Entropy decay for the Kac evolution

Tobias Ried
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We consider solutions to the Kac master equation for initial conditions where N particles are in a thermal equilibrium and $M \leq N$ particles are out of equilibrium. We show that such solutions have exponential decay in entropy relative to the thermal state. More precisely, the decay is exponential in time with an explicit rate that is essentially independent on the particle number. This is in marked contrast to previous results which show that the entropy production for arbitrary initial conditions is inversely proportional to the particle number. The proof relies on Nelson's hypercontractive estimate and the geometric form of the Brascamp-Lieb inequalities due to Franck Barthe. Similar results hold for the Kac-Boltzmann equation with uniform scattering cross sections.

(joint work with Federico Bonetto, Alissa Geisinger, Michael Loss)

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Schrödinger operators exhibiting a sudden change of the spectral character

Pavel Exner
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The aim of this talk is to discuss several classes of Schrödinger operators with potentials that are below unbounded but their negative part is localized in narrow channels. A prototype of such a behavior can be found in Smilansky-Solomyak model devised to illustrate that an an irreversible behavior is possible even if the heat bath to which the systems is coupled has a finite number of degrees of freedom. We review its properties and analyze several modifications of this model, with regular potentials or a magnetic field, as well as another system in which $x^p y^p$ potential is amended by a negative radially symmetric term. All of them have the common property that they exhibit a sudden parameter-dependent spectral transition: if the coupling constant exceeds a critical value the spectrum changes from a below bounded, partly or fully discrete, to the continuous one covering the whole real axis. We also discuss resonance effects in such models. The results come from a common work with Diana Barseghyan, Vladimir Lotoreichik and Miloš Tater.

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The spectral determinant of the quantum harmonic oscillator in arbitrary dimensions

Pedro Freitas
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We show that the spectral determinant of the isotropic quantum harmonic oscillator converges exponentially to one as the space dimension grows to infinity. We determine the precise asymptotic behaviour for large dimension and obtain estimates valid for all cases with the same asymptotic behaviour in the large.

As a consequence, we provide an alternative proof of a conjecture posed by Bär and Schopka concerning the convergence of the determinant of the Dirac operator on S^n, determining the exact asymptotic behaviour for this case and thus improving the estimate on the rate of convergence given in the proof by Møller.

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Eigenvalue inequalities for the Laplacian with mixed boundary conditions

Jonathan Rohleder
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Inequalities for the eigenvalues of the Laplacian subject to mixed boundary conditions on polyhedral and more general bounded domains are established. The eigenvalues subject to a Dirichlet boundary condition on a part of the boundary and a Neumann boundary condition on the remainder of the boundary are estimated in terms of either Dirichlet or Neumann eigenvalues. Moreover, an ordering result for the first mixed eigenvalues of polygons depending on the choice of the Dirichlet part of the boundary is presented. Parts of the results are joint work with Vladimir Lotoreichik.

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Eigenfunctions for versions of the p-Laplacian.

Bernd Kawohl
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The eigenvalue problems $\Delta_p u + \lambda |u|^{p-2} u = 0$ and $\frac{1}{p}|\nabla u|^{2-p}\Delta_p u + \lambda u =$ in Ω under Dirichlet or Neumann boundary conditions and their limits as $p \to 1$ or $p \to \infty$ lead to interesting geometric questions. In this little survey I show how these problems originate from questions in mathematical image processing and what is known and unknown about them.

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Sharp inequalities for eigenvalues and eigenfunctions of some classes of nonlinear elliptic operators

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In this talk I will discuss on sharp bounds for eigenvalues and eigenfunctions of certain classes of nonlinear elliptic operators in bounded domains, under various boundary conditions. The obtained results are contained in some joint papers with F. Della Pietra, G. di Blasio and L. Trani.

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Optimization problems related to some anisotropic operators

Francesco Della Pietra
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In this talk we will describe some recent results on optimal bounds, in terms of quantities which depend on the geometry of the domain, for the extrema of certain functionals which involve an anisotropic norm of the gradient. The results I will describe are obtained in collaboration with N. Gavitone, S. Guarino Lo Bianco and G. di Blasio.

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Rigorous approximation for the density functional of kinetic energy and applications

Phan Thành Nam
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We will discuss the best approximation for the kinetic energy of a many-body fermionic system, in terms of the one-body density functional. This is an old question with many open problems. For the lower bound, we will rigorously derive the sharp semiclassical approximation with an error term of gradient type. For the upper bound, we will construct a Slater determinant which has the matching kinetic energy and an approximate density. Both results are achieved by a closer look of the free Fermi gas in cubes and a localization method. Although the approach is simple, I hope it will initiate some fresh ideas on the old subject. In particular, our results provide a short proof of the validity of the Thomas-Fermi theory for large atoms.

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Fermionic behavior of ideal anyons

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Anyons are magnetically interacting quantum particles in two dimensions which may be considered as intermediate to bosons and fermions. While the former may all assume the lowest energy, say the ground-state eigenvalue of the Laplacian on a domain, the latter must fill higher energy levels, leading to an extensivity according to Weyl's law for the eigenvalues. We prove upper and lower bounds on the ground-state energy of the ideal anyon gas which are extensive in the particle number and linear in the interpolating statistics parameter. The lower bounds extend to Lieb-Thirring inequalities for all anyons except bosons. This is recent joint work with Robert Seiringer.

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Van der Waals Force in Pseudo-Relativistic Molecules

Semjon Wugalter
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The Van der Waals force between atoms and molecules plays an important role in chemistry, physics and biology. It explains different processes and phenomena from condensation of water up to shapes of gigantic molecules such as proteins and DNA.
Mathematically rigorous computation of the Van der Waals interaction energy was given recently by I. Anapolitanos and I.M. Sigal for non-relativistic Schrödinger operators. In the talk, applying a different method, this result will be extended to operators with pseudo-relativistic kinetic energy. In addition to this extension in both relativistic and non-relativistic cases we compute higher order corrections to the Van der Waals-London law.
The talk is based on a joint work with J.-M. Barbaroux, M. Hartig and D. Hundertmark.

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Minimal k-partition for the p-norm of the eigenvalues

Virginie Bonnaillie-Noël
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In this talk,we would like to analyze the connections between the nodal domains of the eigenfunctions of the Dirichlet-Laplacian and the partitions of the domain by k open sets D_i which are minimal in the sense that the maximum over the D_i's of the groundstate energy of the Dirichlet realization of the Laplacian is minimal. Instead of considering the maximum among the first eigenvalues, we can also consider the p-norm of the vector composed by the first eigenvalues of each subdomain.

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On the existence of impurity bound excitons in one-dimensional systems with zero range interactions

Horia Cornean
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We consider a three-body one-dimensional Schrödinger operator with zero range potentials, which models a positive impurity with charge $\kappa > 0$ and infinite mass interacting with a pair of oppositely charged particles having equal masses. We study the existence of discrete eigenvalues as κ is varied. On one hand, we show that for sufficiently small κ there exists a unique bound state whose binding energy behaves like κ^4, and we explicitly compute its leading coefficient. On the other hand, if κ is larger than some critical value, then the system has no bound states. Joint work with J. Have, H. Kovařík and TG Pedersen.

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Sharp boundary behavior of eigenvalues for Aharonov-Bohm operators with varying poles

Benedetta Noris
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In this talk, I will discuss the behavior of the eigenvalues of a magnetic Aharonov-Bohm operator with half-integer circulation and Dirichlet boundary conditions in a bounded planar domain. A sharp relation is established between the rate of convergence of the eigenvalues as the singular pole is approaching a boundary point and the number of nodal lines of the eigenfunction of the limiting problem.

The talk is based on papers in collaboration with L. Abatangelo, V. Bonnaillie-Noël, V. Felli, M. Nys and S. Terracini.

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On Aharonov-Bohm operators with two colliding poles

Veronica Felli
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In this talk, I will discuss spectral stability for magnetic Schrödinger operators of Aharonov-Bohm type. In particular I will consider Aharonov-Bohm operators with two poles and describe the asymptotic behavior for simple eigenvalues as the poles collapse at an interior point.

This talk is based on papers in collaboration with L. Abatangelo (Milano-Bicocca), L. Hillairet (Orléans), and C. Léna (Lisbon).

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Nodal count distribution of graph eigenfunctions

Gregory Berkolaiko
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We start by reviewing the notion of "quantum graph" its eigenfunctions and the problem of counting the number of their zeros. The nodal surplus of the n-th eigenfunction is defined as the number of its zeros minus (n-1). When the graph is composed of two or more blocks separated by bridges, we propose a way to define a "local nodal surplus" of a given block. Since the eigenfunction index n has no local meaning, the local nodal surplus has to be defined in an indirect way via the nodal-magnetic theorem of Berkolaiko, Colin de Verdière and Weyand.

We will discuss the properties of the local nodal surplus and their consequences. In particular, its symmetry properties allow us to prove the long-standing conjecture that the nodal surplus distribution for graphs with β disjoint loops is binomial with parameters (β,1/2).

The talk is based on joint work with Lior Alon and Ram Band, arXiv:1709.10413 (accepted to CMP).

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Inequalities for eigenfunctions of quantum graphs.

Evans Harrell
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I'll discuss ways to construct realistic landscape functions for eigenfunctions ψ of quantum graphs. The term "landscape functions" refers to functions that are easier to calculate than exact eigenfunctions, but which dominate $|\psi|$ in a non-uniform pointwise fashion indenbtifying where ψ can be localized. Different techniques are found to work best in different parts of the graph, depending on the relationship between the eigenvalue and the potential energy V(x); in different regimes we use Agmon's method, a maximum-principle comparison with a simplified torsion function, and some strictly one-dimensional methods.

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3 List of Participants

Participants marked with ※ are speakers, the superscript numbers give the page with the corresponding abstract and further information as well as the page on the schedule.

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