

Geometry and Topology

Counting Problems and Homological Stability

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In 1970, Arnold showed that the i^{th} homology of the space of un-ordered configurations of n points in the plane becomes independent of n for large n . A decade later, Segal extended Arnold's method to show that the i^{th} homology of the space of degree n holomorphic maps from \mathbf{P}^1 to itself also becomes independent of n for large n , and, moreover, that both sequences of spaces have the same limiting homology. We explain how, using Weil's number field/function field dictionary, one might have predicted this topological coincidence from easily verifiable statements about specific counting problems. We then discuss ongoing joint work with Benson Farb and Melanie Wood in which we use other counting problems to predict and discover new homological stability phenomena in the topology of complex manifolds.