THE ROYAL SWEDISH ACADEMY OF SCIENCES
INSTITUT MITTAG-LEFFLER
Auravägen 17, SE-182 60 Djursholm, Sweden
Tel. +46 8 622 05 60 Fax. +46 8 622 05 89
info@mittag-leffler.se www.mittag-leffler.se

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M. Denker and M. Nicol

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Erdös Rényi laws for hyperbolic dynamical systems

Manfred Denker and Matthew Nicol*

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Abstract

We establish Erdös-Rényi limit laws for Lipschitz observations on a class of non-uniformly expanding dynamical systems, including logistic-like maps. These limit laws give the maximal average of a time series over a time window of logarithmic length. We also give results on the rate of convergence in the limit law.

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1 Introduction

The Erdös-Rényi law was first formulated for independent and identically distributed random variables in 1970 ([6]) as follows:

Theorem 1.1 *Let* $(X_n)_{n\geq 1}$ *be an independent identically distributed (iid) sequence of non-degenerate random variables, and put* $S_n = X_1 + ... + X_n$*. Assume that the moment generating function* $\varphi(t) = Ee^{tX_1}$ *exists in some interval* U *containing* $t = 0$*. For each* $\alpha > 0$*, define* $\psi_\alpha(t) = \varphi(t)e^{-\alpha t}$*. For those* α *for which* ψ_α *attains its minimum at a point* $t_\alpha \in U$*, set* $c_\alpha = -1/\log \psi_\alpha(t_\alpha)$*. Then*

$$\lim_{N\to\infty} \max\{(S_{n+[c_\alpha \log N]} - S_n)/[c_\alpha \log N] : 1 \leq n \leq N - [c \log N]\} = \alpha.$$

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To appreciate the significance of the theorem suppose X_i is an iid sequence taking on the values ± 1 with equal probability and define $\theta(N, K(N))$ by

$$\theta(N, K(N)) := \max_{0 \le n \le N-K(N)} \frac{S_{n+K(N)} - S_n}{K(N)}$$

$\theta(N, K(N))$ may be interpreted as the maximal average gain over a time window of length $K(N)$. A straightforward calculation using the strong law of large numbers shows that if $\lim_{N\to\infty} \frac{K(N)}{\log N} = \infty$ then $\lim_{N\to\infty} \theta(N, K(N)) = 0$, P a.s. However if $K(N) \le c \log_2 N$ with $0 < c < 1$ then in the limit of large N with probability one there is at least one $n < N - K(N)$ such that $X_{n+1} = X_{n+2} = \ldots = X_{n+K(N)} = 1$ so that $\lim_{N\to\infty} \theta(N, K(N)) = 1$ P a.s. Thus in this setting of a fair game the Erdös-Rényi law gives information on the maximal average gain of a player in a fair game precisely in the case where the length of the time window ensures $\lim_{N\to\infty} \theta(N, K(N))$ has a non-degenerate limit. As another application, Erdös and Rényi take X_i to be iid with the standard normal distribution $N(0, 1)$ and give a simple proof of a remarkable result of Lévy [8]: if $B(t)$ is canonical Brownian motion then

$$\lim_{t\to 0} P(|B(t+h) - B(t)| < \lambda \sqrt{2h \log \frac{1}{h}} \text{ for } 0 \le t \le 1-h\ ) = \begin{cases} 1 & \text{if } \lambda > 1; \\ 0 & \text{if } \lambda < 1. \end{cases}$$

In this note we establish Erdös-Rényi limit laws for certain non-uniformly expanding maps. We also discuss stronger versions which give rates of convergence. In the dynamical systems context such a result has first been obtained by Grigull [7] in 1993, later by Chazottes and Collet [1] for uniformly expanding maps of the interval, and for Gibbs-Markov dynamics by Denker and Kabluchko [3]. The results of Chazottes and Collet [1] also give a convergence rate (as do Deheuvels et al [5] for the independent case). The convergence rates we give may not be optimal.

2 Erdös-Rényi law

Suppose that $(T, X, \mathcal{B}, \mu)$ is a probability preserving transformation and $\varphi : X \to \mathbb{R}$ is a mean-zero integrable function i.e. $E(\varphi) := \int_X \varphi d\ \mu = 0$. Let $S_n(\varphi) := \varphi + \varphi \circ T + \ldots + \varphi \circ T^{n-1}$.

Definition 2.1 *A mean-zero integrable function $\varphi : X \to \mathbb{R}$ is said to satisfy a large deviation principle with rate function $I(\alpha)$, if there exists a neigh-*

bourhood U of 0 and a strictly convex function $I : U \to \mathbb{R}$, non-negative and vanishing only at $\alpha = 0$, such that

$$\lim_{n\to\infty} \frac{1}{n} \log \mu(S_n(\varphi) \geq n\alpha) \quad = \quad -I(\alpha) \tag{1}$$

for all $\alpha > 0$ in U and

$$\lim_{n\to\infty} \frac{1}{n} \log \mu(S_n(\varphi) \leq n\alpha) \quad = \quad -I(\alpha) \tag{2}$$

for all $\alpha < 0$ in U.

The rate function $I(\alpha)$ is also called the information function. Throughout this paper we will concentrate on the case $\alpha > 0$ as the case $\alpha < 0$ is identical with the obvious modifications of statements.

The first result is well known and may be established by an adapted proof from Grigull [7] or Denker and Kabluchko [3], for example. We give the proof for completeness.

Proposition 2.2 *Suppose that φ satisfies a large deviation principle with rate function I defined on the open set U. Let $\alpha > 0$ and set*

$$l_n = l_n(\alpha) = \left[\frac{\log n}{I(\alpha)}\right] \qquad n \in \mathbb{N}.$$

Then the upper Erdös-Rényi law holds, that is

$$\limsup_{n\to\infty} \max\{S_{l_n}(\varphi) \circ T^j / l_n : 0 \leq j \leq n - l_n\} \leq \alpha.$$

Moreover, if for some constant $C > 0$ and integer $\tau \geq 1$ for each interval A

$$\mu(\bigcap_{m=0}^{n-l_n} \{S_{l_n}(\varphi) \circ T^m \in A\}) \leq C[\mu(S_{l_n} \in A)]^{n/(l_n)^\tau}$$

then the lower Erdös-Renyi law holds as well, i.e.

$$\liminf_{n\to\infty} \max\{S_{l_n}(\varphi) \circ T^j / l_n : 0 \leq j \leq n - l_n\} \geq \alpha.$$

Remark 2.3 *Proposition 2.2 implies that*

$$\lim_{n\to\infty}[\max_{0\le m\le n-l_n}\frac{S_{l_n}\circ T^m}{l_n}]=\alpha$$

Proof. Let $\alpha\in U$ and l_n be as above. We consider the case $\alpha>0$. To simplify notation we write $S_n(\varphi)$ as S_n.

Choose $\epsilon>0$ such that $\alpha+\epsilon\in U$, and define

$$A_n(\epsilon)=\{x\in X:\max_{0\le m\le n-l_n}S_{l_n}\circ T^m\ge(\alpha+\epsilon)l_n\}.$$

If $0<2\delta<I(\alpha+\epsilon)-I(\alpha)$, then there exists $N\in\mathbb{N}$ such that for $n>N$

$$\begin{aligned}\mu(A_n(\epsilon)) &\le n\mu(S_{l_n}\ge(\alpha+\epsilon)l_n)\\ &\le ne^{-l_n(I(\alpha+\epsilon)-\delta)}\\ &\le ne^{-l_n(I(\alpha)+\delta)}\\ &\le e^{I(\alpha)+\delta}n^{-\frac{\delta}{I(\alpha)}}.\end{aligned}$$

Apply this estimate for $n=k^p$ where $p>\frac{I(\alpha)}{\delta}$ is an integer to obtain via the Borel-Cantelli lemma that

$$\limsup_{k\to\infty}\max_{0\le m\le k^d-l_{k^d}}S_{l_{k^d}}\circ T^m/l_{k^d}\le\alpha+\epsilon.$$

Replacing l_n by $l'_n=l_n-1$ yields

$$\limsup_{k\to\infty}\max_{0\le m\le k^d-l'_{k^d}}S_{l_{k^d}}\circ T^m/l_{k^d}\le\alpha+\epsilon.$$

Now take any $n\in\mathbb{N}$. Choose k such that $(k-1)^d<n\le k^d$. Then, for k large, $l_{(k-1)^d}$ and l_{k^d} differ by at most one. Hence $l_n=l_{k^d}$ or $l_n=l'_{k^d}$ and therefore

$$S_{l_n}\circ T^m=S_{l_{k^d}}\quad\text{or}\quad S_{l_n}\circ T^m=S_{l'_{k^d}}\circ T^m$$

for $0\le m\le n$. This shows that

$$\limsup_{n\to\infty}\max_{0\le m\le n-l_n}S_{l_n}\circ T^m/l_n\le\alpha+\epsilon.$$

Letting $\epsilon \to 0$ proves the first part of the proposition.

For the converse inequality, let $\alpha > 0$ and choose $\epsilon > 0$ such that $\alpha - \epsilon > 0$. Define

$$B_n(\epsilon) = \{\max_{0 \le m \le n - l_n} S_{l_n} \circ T^m \le l_n(\alpha - \epsilon)\}.$$

Define also

$$C_m(\epsilon) = \{S_{l_n} \circ T^m \le l_n(\alpha - \epsilon)\}.$$

Then $B_n(\epsilon) = \bigcap_{m=0}^{n-l_n} C_m(\epsilon)$ and by assumption

$$\mu(B_n(\epsilon) \le C\mu(S_{l_n} \le l_n(\alpha - \epsilon)))^{n/(l_n)^\tau}.$$

Using the large deviation property for $C_0(\epsilon)^c := X \setminus C_0(\epsilon)$, for large n one obtains $\mu(C_0(\epsilon)^c) \ge e^{-l_n(I(\alpha-\epsilon)+\delta_1)} \ge e^{-(\frac{I(\alpha-\epsilon)}{I(\alpha)} \log n} e^{I(\alpha-\epsilon)+\delta_1 \frac{\log n}{I(\alpha)}}$. For large n (note $\delta_1 \to 0$ with n) one obtains

$$1 - \mu(C_0(\epsilon)) \ge e^{-(1-\delta)\log n}.$$

for some $0 < \delta < 1$. Therefore

$$\begin{aligned} \mu(B_n(\epsilon)) &\le C\left[1 - e^{-(1-\delta)\log n}\right]^{n/(l_n)^\tau} \\ &= O(\exp -n^{\delta'}). \end{aligned}$$

where δ' is any $0 < \delta' < \delta$. The lower bound follows from the Borel-Cantelli lemma.

Remark 2.4 *It is clear from the proof that to obtain the upper Erdös-Rényi law it suffices to have exponential large deviations given by a rate function, while for the lower Erdös-Rényi law it suffices to show that for every $\epsilon > 0$ the series $\sum_{n>0} \mu(B_n(\epsilon))$ is summable.*

3 One-dimensional non-uniformly expanding maps.

We now give some applications of Proposition 2.2. Melbourne and Nicol [10] have established the existence of a rate function $I(.)$ for a broad class of non-uniformly expanding maps and non-uniformly hyperbolic systems (see also [11] for related results). For these systems the upper Erdös-Renyi law of Proposition 2.2 immediately holds for Lipschitz functions. In this section we verify the mixing condition of Proposition 2.2 to establish the lower Erdös-Renyi law for Lipschitz functions for a class of non-uniformly expanding maps. Our results in this section rely on the existence of an exponential rate function, exponential decay of correlations and a bounded derivative for T.

Theorem 3.1 *Suppose that $T : X \to X$ is a C^2 non-uniformly expanding map of the interval X with an absolutely continuous invariant probability measure μ satisfying exponential decay of correlations of the form: for all $\varphi \in Lip$, $\psi \in L^\infty$ we have*

$$|\int \varphi\psi \circ T^j \, dm - \int \varphi \, dm \int \psi \, dm| \leq C\theta^j \|\varphi\|_{Lip} \|\psi\|_\infty$$

where $0 < \theta < 1$ and C is a constant independent of φ and ψ.

Then any Lipschitz observation $\varphi : X \to \mathbb{R}$ has exponential deviations with a rate function and the upper and lower Erdös-Rényi laws of Proposition 2.2 hold.

Remark 3.2 *(i) Theorem 3.1 applies to the class of logistic maps $T(x) = 1 - ax^2$ with a in the set of parameters which lead to an absolutely continuous invariant measure and exponential decay of correlations.*

(ii) Theorem 3.1 also applies to the class of non-uniformly expanding maps of the interval modeled by a Young Tower with exponential decay of correlations considered by Collet [2].

Proof:

Melbourne and Nicol [10, Theorem 2.1] have established the existence of a rate function $I(.)$ for this class of maps. The existence of a rate function implies that the upper Erdös-Rényi law of Proposition 2.2 holds. We will show that the lower Erös-Rényi law of Proposition 2.2 also holds by showing that for every $\epsilon > 0$, $\sum_{n>0} \mu(B_n(\epsilon))$ is summable (see Remark 2.4). To simplify notation we write S_n for $S_n(\varphi)$.

Since T is C^2, $|DT| < L$ for some $L > 0$. For $s > 0$ define $A_n^s = \{S_{l_n} \leq l_n(\alpha - s)\}$. We fix $\epsilon > 0$ and consider $A_n^{\epsilon/2} = \{S_{l_n} \leq l_n(\alpha - \epsilon/2)\}$ and $A_n^\epsilon = \{S_{l_n} \leq l_n(\alpha - \epsilon)\}$.

Let $0 < \eta < 1$. If $|x-y| < L^{-(1+\eta)m}$ then $|T^m x - T^m y| < L^{-\eta m}$ from the mean-value theorem applied to T^m and the fact that $|DT^m| < L^m$. Thus if n is large then $x \in A_n^\epsilon$ and $|x-y| < L^{-(1+\eta)l_n}$ implies that $y \in A_n^{\epsilon/2}$.

We may approximate the indicator function $1_{A_n^\epsilon}$ of A_n^ϵ by a Lipschitz function φ_ϵ of Lipschitz norm at most $L^{(1+\eta)l_n}$ satisfying $1_{A_n^\epsilon} \le \varphi_\epsilon \le 1$ and $\mu(A_n^\epsilon) < \int \varphi_{A_\epsilon} d\mu < \mu(A_n^{\epsilon/2})$. To do this let $F := A_n^\epsilon$ and define $h_1(z) = 0$ and $h_2(z) = 1 - d(z,F)L^{-(1+\eta)l_n}$. The fact that h_2 is Lipschitz with Lipschitz constant $L^{(1+\eta)l_n}$ is straightforward (see for example Stein [12, Section 2.1]). Thus $\varphi_\epsilon(z) := \max\{0, (1-d(z,F)L^{-(1+\eta)l_n}\}$ is Lipschitz with Lipschitz constant bounded by $L^{(1+\eta)l_n}$ and support in $A_n^{\epsilon/2}$.

Define $C_m(\epsilon) = \{S_{l_n} \circ T^m \le l_n(\alpha - \epsilon)\}$ and $B_n(\epsilon) = \bigcap_{m=0}^{n-l_n} C_m(\epsilon)$. We use a blocking argument to take advantage of decay of correlations and intercalate by blocks of length $(\log n)^\tau$, $\tau > 1$. We define

$$E_n^0(\epsilon) := \bigcap_{m=0}^{[(n-(\log n)^\tau)/(\log n)^\tau]} C_{m[(\log n)^\tau]}(\epsilon)$$

and in general for $0 \le j < [\frac{n}{(\log n)^\tau}]$

$$E_n^j(\epsilon) := \bigcap_{m=0}^{[(n-(j+1)(\log n)^\tau)/(\log n)^\tau]} C_{m[(\log n)^\tau]}(\epsilon)$$

Note that $\mu(B_n(\epsilon)) \le \mu(E_n^0(\epsilon))$. For each j, let ψ_j denote the indicator function of $E_n^j(\epsilon)$, so that

$$\psi_j = 1_{E_n^j(\epsilon)}$$

By decay of correlations we have

$$\begin{aligned} \mu(E_n^0(\epsilon)) &\le \int \varphi_\epsilon \psi_1 \circ T^{[(\log n)^\tau]} d\mu \\ &\le C\theta^{(\log n)^\tau} \|\varphi_\epsilon\|_{Lip} \|\psi_1\|_\infty + \int \varphi_\epsilon \, d\mu \int \psi_1 \, d\mu \\ &\le \int \varphi_\epsilon \, d\mu \int \psi_1 \, d\mu + C\theta^{(\log n)^\tau}(L^{(1+\eta)l_n}) \end{aligned}$$

Applying a Lipschitz approximation and decay of correlations again to $\int \psi_1 \, d\mu$ we iterate and conclude

$$\mu(E_n^0(\epsilon)) \le nC\theta^{(\log n)^\tau} L^{(1+\eta)l_n} + \mu(A_n^{\epsilon/2})^{n/(\log n)^\tau}$$

The term $nC\theta^{(\log n)^\tau}L^{(1+\eta)l_n}$ is clearly summable if $\tau > 1$. The same argument given in the proof of Proposition 2.2 using large deviations shows that $\mu(A_n^{\epsilon/2})^{n/(\log n)^\tau}$ is summable for all $\tau \geq 1$ as $\mu(A_n^{\epsilon/2}) \leq 1 - e^{\log n[\frac{I(\alpha-\epsilon)}{I(\alpha)}+\delta_1]}$ where $\delta_1 \to 0$ as $n \to \infty$ so that $\mu(A_n^{\epsilon/2}) \leq 1 - e^{\log n(1-\rho)}$ for some $0 < \rho < 1$. Since the right hand side is summable for any $\tau > 1$ the Borel-Cantelli lemma implies the result by Remark 2.4.

4 Rates of Convergence

In this section we refine our results to consider the rate of convergence in the Erdös-Rényi limits laws. In order to obtain rates in the Erdös-Rényi law we need stronger assumptions. In the setting of topologically mixing piecewise C^2 expanding maps of the interval (T, I, μ) Chazottes and Collet [1, Appendix A] show:

Proposition 4.1 *[1] If $\varphi : I \to \mathbb{R}$ is of bounded variation then*

$$\limsup_{n\to\infty} \max_{0\leq m\leq n-l_n} \frac{S_{l_n}\circ T^m - l_n\alpha}{\log l_n} \leq \frac{1}{2I'(\alpha)} \quad a.e.$$

and

$$\liminf_{n\to\infty} \max_{0\leq m\leq n-l_n} \frac{S_{l_n}\circ T^m - l_n\alpha}{\log l_n} \geq -\frac{1}{2I'(\alpha)} \quad a.e.$$

To prove this they use the following proposition [1, Appendix A], once again in the setting of topologically mixing piecewise C^2 expanding maps of the interval (T, I, μ). Such maps have exponential large deviations estimates given by a rate function, $I(.)$.

Proposition 4.2 *[1] If $\varphi : I \to \mathbb{R}$ is of bounded variation then there is a compact interval K of the origin and constants $0 < c_1 < c_2$ such that for any $\alpha \in K/\{0\}$ and integer $n \geq 1 + I'(\alpha)^{-4}$,*

$$\frac{c_1}{I'(\alpha)\sqrt{n}}e^{-nI(\alpha)} \leq \mu(S_n \geq n\alpha) \leq \frac{c_2}{I'(\alpha)\sqrt{n}}e^{-nI(\alpha)}$$

We now state our rate theorem,

Theorem 4.3 *In addition to the assumptions in Proposition 2.2 we assume that the information function $I(\cdot)$ is twice differentiable on U and the large deviation property holds uniformly in neighborhoods of α for each $\alpha \in U$: There exists $C \geq 1$ and $\kappa(\varphi) \geq 0$ such that for $\alpha \in U$*

$$(*) \;\; \lim_{n\to\infty} \frac{1}{\log n}\left[\log m(S_n \geq \alpha)e^{nI(\alpha)}\right] = \kappa(\varphi)$$

If $\alpha \in U$ then

$$\limsup_{n\to\infty} \max_{0\leq m\leq n-l_n} \frac{S_{l_n}\circ T^m - l_n\alpha}{\log l_n} \leq \frac{1+\kappa}{I'(\alpha)} \quad a.e.$$

and

$$\liminf_{n\to\infty} \max_{0\leq m\leq n-l_n} \frac{S_{l_n}\circ T^m - l_n\alpha}{\log l_n} \geq -\frac{\tau+1+\kappa}{I'(\alpha)} \quad a.e.$$

where τ is defined in Proposition 2.2.

Proof:

Let $d > \frac{1+\kappa}{I'(\alpha)}$ and choose $\delta > 0$ with $dI'(\alpha) - \kappa - 2\delta > 1$. We have for all n large enough

$$\begin{aligned}
& \mu\left(\max_{0\leq m\leq n-l_n} S_{l_n}\circ T^m - l_n\alpha \geq d\log l_n\right) \\
\leq\; & n\mu\left(S_{l_n} \geq l_n(\alpha + d\frac{\log l_n}{l_n})\right) \\
\leq\; & I(\alpha)^{-\kappa-\delta} n l_n^{\kappa+\delta} \exp\left[-l_n\left(I\left(\alpha + d\frac{\log l_n}{l_n}\right)\right)\right] \\
\leq\; & I(\alpha)^{-\kappa-\delta} n l_n^{\kappa+\delta} \exp\left[-l_n I(\alpha) - I'(\alpha)d\log l_n + \frac{1}{2}I''(\zeta)d^2\frac{(\log l_n)^2}{l_n}\right] \\
=\; & O\left(l_n^{-I'(\alpha)d+\kappa+2\delta}\right).
\end{aligned}$$

Since $I'(\alpha)d - \kappa - 2\delta > 1$, it follows from the Borel-Cantelli Lemma, applied to the subsequence $q_k = [q^k]$ $(k \geq 1)$ where $q > 1$, that

$$\limsup_{k\to\infty} \max_{0\leq m\leq q_k - l_{q_k}} (S_{l_{q_k}}\circ T^m - l_{q_k}\alpha)/\log l_{q_k} \leq d. \tag{3}$$

We now proceed as in the proof of Proposition 2.2. Note that $l_{q^{k+1}} - l_{q^k}$ is bounded by a constant $\leq \log q + 1$. Therefore we can use the equation (3) replacing l_{q^k} by $l_{q^k} - j$ for $j = 0, 1, 2, ..., [1 + \log q]$.

It follows that

$$\limsup_{n\to\infty} \max_{0\leq m\leq n-l_n} (S_{l_n} \circ T^m - l_n\alpha)/\log l_n \leq d.$$

For the converse, define

$$B_n = \{\max_{0\leq m\leq n-l_n} S_{l_n} \circ T^m \leq l_n\alpha - d\log l_n\},$$

and

$$C_m = \{S_{l_n} \circ T^m \leq l_n\alpha - d\log l_n\} \qquad m = 0, ..., n - l_n.$$

Then $B_n = \bigcap_{m=0}^{n-l_n} C_m$ and by assumption

$$\mu(B_n) \leq C[\mu(S_{l_n} \leq l_n\alpha - d\log l_n)]^{n/(l_n)^\tau}.$$

By the large deviation rate estimate $(*)$

$$\begin{aligned}
1 - \mu(S_{l_n} \geq 1 - l_n\alpha - d\log l_n) &\geq l_n^\kappa \exp\left[-l_n I\left(\alpha - d\frac{\log l_n}{l_n}\right)\right] \\
&= 1 - l_n^\kappa \exp\left[-l_n\left(I(\alpha) - I'(\alpha)d\frac{\log l_n}{l_n} + \frac{1}{2}I''(\zeta)d^2\frac{(\log l_n)^2}{l_n^2}\right)\right] \\
&\geq 1 - n^{-1}l_n^\kappa \exp[I'(\alpha)d\log l_n - \delta_n \log l_n] \\
&= 1 - n^{-1}l_n^{I'(\alpha)d+\kappa-\delta_n}
\end{aligned}$$

where $\lim_{n\to\infty}\delta_n = 0$.

It follows that

$$\begin{aligned}
\mu(B_n) &\leq C\left(1 - n^{-1}l_n^{I'(\alpha)d+\kappa-\delta_n}\right)^{n/(l_n)^\tau} \\
&= O\left(\exp\left[-l_n^{I'(\alpha)d+\kappa-1-\delta_n}\right]\right),
\end{aligned}$$

If $d > \frac{\kappa+\tau+1}{I'(\alpha)}$ then $\sum_n \mu(B_n) < \infty$ and by the Borel Cantelli lemma we arrive at

$$\liminf \max_{0\leq m\leq n-l_n} (S_{l_n} \circ T^m - l_n\alpha)/\log l_n \geq -d.$$

5 Applications of Theorem 4.3

Theorem 5.1 *Suppose that $T : X \to X$ is a subshift of finite type with the usual symbolic metric and m is a Gibbs state corresponding to a Hölder potential. Then the conclusions of Theorem 4.3 hold (with $\tau = 2$) for any Lipschitz observation $\varphi : X \to \mathbb{R}$.*

Proof:

The same proof given for C^2 non-uniformly expanding maps of the interval X with exponential decay of correlations holds in this setting to establish the assumptions of Proposition 2.2 (we may take $\tau = 2$). The existence of a $\kappa(\varphi)$ such that for $\alpha > 0$

$$\frac{1}{\log n}\left[\log m(S_n \geq \alpha)e^{nI(\alpha)}\right] = \kappa(\varphi)$$

follows from a result of Kessebohmer [9, Corollary 5.4] on large deviations for subshifts of finite type.

References

[1] Chazottes, J.-R.; Collet, P.: Almost-sure central limit theorems and the Erdös-Rényi law for expanding maps of the interval. Ergodic Theory Dynam. Systems 25 (2005), 419–441.

[2] Collet, P: Statistics of closest return for some non-uniformly hyperbolic systems, Erg. Th. Dyn. Syst., 21, (2001), 401-420.

[3] Denker, M.; Kabluchko, Z.: An Erdös-Rényi law for mixing processes. Probab. Math. Statist. 27 (2007), 139–149.

[4] Denker, M.: Large deviations and the pressure function. In: Transactions 11th Prague Conference on Information Theory, Statistical Decision Functions, Random Processes 1990. Academia Publ. House of the Czechoslovak Academy of Sciences, 1992, 21–33.

[5] Deheuvels, P.; Devroye, L.; Lynch, J.: Exact convergence rate in the limit theorems of Erdös-Rényi and Shepp. Ann. Probab. 14 (1986), 209–223.

[6] Erdös, P.; Rényi, A.:: On a new law of large numbers. J. Analyse Math. 23 (1970), 103–111.

[7] Grigull, J.: Große Abweichungen und Fluktuationen für Gleichgewichtsmaße rationaler Abbildungen. Dissertation Göttingen University 1993.

[8] P. Lévy, Théorie de l'addition des variables aléatoires indépendantes, Paris, Gauthier-Villars.

[9] M.; Kessebohmer: Multifraktale und Asymptotiken grosser Deviationen", Dissertation, GeorgAugustUniversität zu Göttingen 1999

[10] Melbourne, I.; Nicol, M.: Large deviations for nonuniformly hyperbolic systems. Trans. Amer. Math. Soc. 360 (2008), 6661–6676.

[11] Rey-Bellet,L; Young,L.-S: Large deviations in nonuniformly hyperbolic dynamical systems. Ergodic Theory and Dynamical Systems 28, (2008) 587-612

[12] Stein, E: Singular integrals and differentiability properties of functions, Princeton University Press, 1986.

Authors' addresses:
Denker: Mathematics Department, Pennsylvania State University, State College PA 16802,
USA
e-mail: denker@math.psu.edu
Nicol: Department of Mathematics, University of Houston, Houston TX 77204-3008,
USA
e-mail: nicol@math.uh.edu