

Logic

When the principle of omniscience just holds

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Without assuming excluded middle, in constructive mathematics one may prove that plenty of infinite sets X satisfy the principle of omniscience, which gives a logical counter-part of the topological notion of compactness, generalizing constructive notions of finiteness. The principle says that it is decidable whether any given 2-valued function on X has a zero or not. We examine this in the context of Martin-Löf type theory, touching some concepts of Univalent Foundations (UF). In particular, we discuss the role of two different notions of constructive existence present in UF. We provide new methods of constructing omniscient sets, involving the injectivity of the universe of types and a certain "micro Tychonoff Theorem" that asserts the closure of omniscient types under products indexed by a subsingleton. The omniscient types we obtain turn out to be ordinals with the property that every non-empty decidable subset has a least element. They are countable in a weak sense.