Logic

Shelah's Main Gap Theorem in the Borel-reducibility hierarchy

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One way of classifying first-order theories is using Shelah's stability theory. One of the main results is the Main Gap Theorem. This theorem tells us that classifiable theories are less complex than the non-classifiable ones and also that their complexity classes are far apart.

Another way of classifying first-order theories is using descriptive set theory, namely Borel-reducibility. When we use classical descriptive set theory (i.e. studying models of countable cardinality), we do not get the same re-sults as with stability theory: sometimes descriptive set theory gives trivial complexity to theories that are non-trivial in stability theoretic sense and vice versa.

However, if we use generalized descriptive set theory (i.e. studying models of uncountable cardinality), we can see a connection between generalized descriptive set theory and stability theoretic approaches. One question that arises is: Is there a counterpart of Shelah's main gap theorem in the context of Borel-reducibility?

In generalized descriptive set theory, we work in the generalized Baire space κ^κ (κ an uncountable cardinal) and the complexity of a theory is measured by determining the place of the isomorphism relation of models of size κ in the Borel-reducibility hierarchy. The different kind of theories (classifi-able, unstable, strictly stable, etc) have a total or partial characterization in the Borel-reducibility hierarchy so far (see the work of Friedman, Hyttinen, Kulikov, etc.).

Employing the equivalence modulo non-stationary ideals, we will see that: It is consistent that this equivalence relation is strictly above the isomorphism of every classifiable, and strictly below the isomorphism of every non-classifiable theory, with respect to Borel-reducibility. It is also consistent that for every classifiable and every non-classifiable theory we can embed the partial order $(P(\kappa), \subset)$ into the Borel-reducibility partial order strictly between the isomorphism relations of these theories.

(This is joint work with T. Hyttinen and V. Kulikov)