Logic

"Type theory as a language for homotopy theory": what does it mean, really?

Peter LeFanu Lumsdaine

Department of Mathematics, Stockholm University, Sweden

Around ten years ago, the discovery of rich connections between type theory and homotopy-theoretic settings (by Hoffmann–Streicher, Voevodsky, Awodey–Warren, and others) sparked off the programme of work known as homotopy type theory or univalent foundations.

Since then, slogans like "type theory is the logic of homotopy theory" and "homotopy type theory is the internal language of infinity-toposes" have been widely bandied around; but their precise status has not always been clear.

I will survey the different things these could or should mean — some heuristic, some precise; some conjectural, some now established. This will include (among other things) recent work of Kapulkin, Shulman, Cisinski, and myself.