A note on some fiber-integrals $\int_{f=s} \rho.(\omega/df) \wedge \overline{(\omega/df)}$

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We remark that the study of a fiber-integral of the type

$$F(s) := \int_{f=s} \rho.(\omega/df) \wedge \overline{(\omega/df)}$$

either in the local case where $\rho \equiv 1$ around 0 is $\mathcal{C}^\infty$ and compactly supported near the origin which is a singular point of $\{f = 0\}$ in $\mathbb{C}^{n+1}$, or in a global setting where $f : X \to D$ is a proper holomorphic function on a complex manifold X, smooth outside $\{f = 0\}$ with $\rho \equiv 1$ near $\{f = 0\}$, for given holomorphic $(n+1)-$forms ω and ω', that a better control on the asymptotic expansion of F when $s \mapsto 0$, is obtained by using the Bernstein polynomial of the "frescos" associated to f and ω and to f and ω' (a fresco is a "small" Brieskorn module corresponding to the differential equation deduced from the Gauss-Manin system of f at 0) than to use the Bernstein polynomial of the full Gauss-Manin system of f at the origin. We illustrate this in the local case in some rather simple (non quasi-homogeneous) polynomials, where the Bernstein polynomial of such a fresco is explicitly evaluated.