A characterization of polynomials in complex and non-archimedean dynamics

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In 1960s Hans Brolin initiated systematic application of potential-theoretic methods in the dynamics of holomorphic polynomials. Among other things, he proved the now-famous equidistribution theorem: for a polynomial f of degree greater than 1 the preimages, under successive iterates of f, of a Dirac measure at an arbitrary point of the complex plane (except at most two so-called exceptional points) converge weakly to the equilibrium mea-sure of the Julia set for f. In 1980s a similar result (about convergence of preimages of quite general probabilistic measures) was proved for a rational map f of degree greater than 1. The limit measure obtained in this case (called the balanced measure) is also supported on the Julia set for f, but does not have to be its equilibrium measure. In fact, A.O. Lopes proved (us-ing dynamical properties of Julia sets) that equality of these two measures (under suitable assumptions on f, also making precise the notion of the equi-librium measure for the Julia set) implies that f is a polynomial. In 2010 we obtained a proof of Lopes's theorem (under slightly weaker assumptions) using only classical and weighted potential theory. In this talk I will present a recent extension of this result, namely a characterization of polynomials among rational functions, up to rational functions having potentially good reductions as exceptions, on the projective line over an algebraically closed field of any characteristic that is complete with respect to a non-trivial and possibly non-archimedean absolute value. I will introduce basic notions of non-archimedean dynamics and discuss possible cases. This is joint work with Yusuke Okuyama from Kyoto Institute of Technology.