

# Number theory

## Algebraicity of automorphic representations

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One striking implication of Langlands' conjectures for number fields is that many automorphic representations which are initially defined by analytic and/or representation-theoretic means should have deep algebro-geometric properties, ranging from the algebraicity of Hecke eigenvalues, to the existence of associated Galois representations and ultimately pure motives. An example of a long-standing open problem in this area which admits an elementary formulation is to prove that Maass forms of eigenvalue  $1/4$  have algebraic Hecke eigenvalues. Two approaches have been used to verify Langlands' predictions: (1) Finding and exploiting a direct link with algebraic geometry and (2) Using Langlands' Functoriality Principle. I will discuss the possibilities and limitations of the two approaches and report on recent work on each approach. The results using the geometric approach are joint work with Jean-Stefan Koskivirta, see [arXiv:1507.05032](https://arxiv.org/abs/1507.05032).