

PDE session

Can a crack form an infinite spiral?

John Andersson

KTH, Sweden

The Mumford-Shah functional is

$$J(u, \Gamma) = \int_{B_1(0) \setminus \Gamma} |\nabla u(x)|^2 dx + \mathcal{H}^1(\Gamma),$$

where Γ is an unknown set and \mathcal{H}^1 is the Hausdorff measure. The minimizer is a pair (u, Γ) where $u \in W^{1,2}(B_1(0) \setminus \Gamma)$.

It is well known that if (u, Γ) is a minimizing pair then Γ is $C^{1,\alpha}$ regular a.e. (w.r.t. \mathcal{H}^1). In this talk we will show that Γ is $C^{1,\alpha}$ regular close to its end points. The results are new and are not covered by the previous work on the Mumford-Shah problem. As a corollary to our method it follows that if (u, Γ) is a minimizer then the curvature vanishes at the end points of the crack.

The problem has applications in fracture theory where Γ models the crack in a solid. It is of importance for the theory of quasi static crack growth to know the behaviour of the crack close to the crack tip.