

PDE session

Some Non-linear Eigenvalue Problems and Rayleigh Quotients

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Old and some new results about certain non-linear eigenvalue problems will be discussed. The equation

$$\nabla \cdot (|\nabla u|^{p-2} u) + \lambda |u|^{p-2} u = 0$$

will appear. Here $u \in W_0^{1,p}(\Omega)$. It turns out, somewhat surprisingly, that the Rayleigh quotient

$$\inf \frac{\int_{\Omega} |\nabla \phi|^2 dx}{\int_{\Omega} |\phi|^2 dx}$$

is unstable under variations of the exponent 2 in the integrands. (The infimum is taken over all smooth functions $\phi \not\equiv 0$ with compact support in the bounded domain Ω .) I shall discuss this interesting instability phenomenon. Some related non-linear eigenvalue problems will be considered. Difficult open problems remain.

PETER LINDQVIST: *Three Nonlinear Eigenvalue Problems*, Oberwolfach Reports **10**, No. 1 (2013), pp. 456–458.