PDE session

A new approach to Sobolev spaces in metric measure spaces?

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The aim of this talk is to discuss a possible alternative way to introduce first order Sobolev spaces in a metric measure space through a type of mass transport metric.

For a suitable metric measure space (X, d_X, μ) we define $\mathcal{X}$ to be the set of all positive, finite non-zero regular Borel measures with compact support in X which are dominated by μ, and $\mathcal{M} = \mathcal{X} \cup \{0\}$. By introducing a kind of mass transport metric $d_{\mathcal{M}}$ we first introduce a Sobolev space for functions $F : \mathcal{X} \to \mathbb{R}$, and then for functions $f : X \to [-\infty, \infty]$ by identifying them with the unique element $F_f : \mathcal{X} \to \mathbb{R}$ defined by the mean-value integral:

$$F_f(\eta) = \frac{1}{\|\eta\|} \int f \, d\eta.$$

At-lest in an open subset of $\mathbb{R}^n$ with Lebesgue measure the approach gives us the classical Sobolev spaces. However what happens in more general situations is not known at present.