PDE session

Nonassociative algebras and nonlinear PDEs

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Minimal cones arise as singular solutions of the 1-Laplace equation

$$\Delta_1 u(x) = |Du|^2 \Delta u - \frac{1}{2} Du \cdot D|Du|^2 = 0.$$

They played the crucial role in both the solution of the famous Bernstein problem and in the construction of the higher dimensional counterexamples by Bombieri, de Giorgi and Giusti [1]. Quadratic minimal cones were completely classified by Hsiang [2]. Despite the fact that the minimal cones of degree 3 and higher have been the subject of considerable recent interest, there is still very little known about their structure and their classification is a long-standing open problem. In my talk, I discuss a recent progress in classification of the so-called Hsiang cubic minimal cones, i.e. the cubic homogeneous solutions of

$$\Delta_1 u(x) = \lambda |x|^2 u(x), \quad \lambda \in \mathbb{R}, \tag{1}$$

by using a nonassociative algebra approach. A remarkable and very surprising result is that any solution of (1) is necessarily *harmonic*, i.e. $\Delta u = 0$, and furthermore, satisfies the Hessian type equation

$$\operatorname{trace}(D^2 u)^3 = 3\lambda(k-1)u, \qquad k \in \mathbb{Z}^+, \tag{2}$$

where k is the dimension of a hidden Clifford algebra representation associated to $u(x)$ in an intrinsic way. Our method is outlined in [3] and [4] and goes back to the Freudenthal-Springer construction in Jordan algebra theory. Some recent applications of cubic minimal cones and nonassociative algebras to constructing truly viscosity solutions of fully nonlinear elliptic PDEs [3] and polynomial solutions of p-Laplace equation [5] will also be discussed.

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References

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