Probability and Statistics

An epidemic in a dynamic population with importation of infectives

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Consider a large uniformly mixing dynamic population, which has constant birth rate and exponentially distributed lifetimes, with mean population size n. A Markovian SIR (susceptible $\rightarrow$ infective $\rightarrow$ recovered) infectious disease, having importation of infectives, taking place in this population is analysed. The main situation treated is where $n \rightarrow \infty$, keeping the basic reproduction number R_0 as well as the importation rate of infectives fixed, but assuming that the quotient of the average infectious period and the average lifetime tends to 0 faster than $1/\log n$. It is shown that, as $n \rightarrow \infty$, the behaviour of the 3-dimensional process describing the evolution of the fraction of the population that are susceptible, infective and recovered, is encapsulated in a 1-dimensional regenerative process $S = \{S(t);\, t \geq 0\}$ describing the limiting fraction of the population that are susceptible. The process S grows deterministically, except at one random time point per regenerative cycle, where it jumps down by a size that is completely determined by the waiting time since the previous jump. Properties of the process S, including the jump size and stationary distributions, are determined.

This is joint work with Frank Ball and Pieter Trapman.