Interfaces between Geometric Analysis and Mathematical Physics

Mittag Leffler conference 7-11. May 2018

Programme schedule and abstracts

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-9:50	Burban	Piazza	Zelditch	Schrohe	Müller
10:00-10:50	Møller	Kröncke	Klevtsov	Strohmeier	Fedosova
Coffee break					
11:30-12:20	Sorensen	Rowlett	Baskin	Lesch	Brüning
lunch					
14:00-14:50	Ma	Cattaneo	(free afternoon)	Waterstraat	
15:00-15:50	Savale	Petersen		Zhu	
Coffee break					
16:30-17:20	Nielsen	Güneysu			
Evening: from 19:00		(Conference dinner)		(Informal talk by Booss)	

Titles and Abstracts:

Dean Baskin (Texas)

Title: Radiation fields on asymptotically Minkowski spacetimes

Abstract: Radiation fields are rescaled limits of solutions of wave equations near "null infinity" and capture the radiation pattern seen by a distant observer. They are intimately connected with the Fourier and Radon transforms and with scattering theory. We find an asymptotic expansion for the radiation field on asymptotically Minkowski spacetimes (as well as on product cones) and show that the exponents seen in the expansion are the resonances of the Laplacian on an associated asymptotically hyperbolic manifold. This talk is based on joint work with Andras Vasy, Jared Wunsch and Jeremy Marzuola.

Bernhelm Booss-Bavnbek (Roskilde)

Title (evening presentation): Science and liability. Criminal prosecution of symbol processing professionals

Abstract. In recent years, we have witnessed a series of criminal prosecutions of symbol processing professionals like seismologists, accountants, programmers, and financial traders, who had played a decisive role in disasters affecting large groups of people. They received severe sentences (between 2 and 11 years of prison), although none of them were prosecuted for commanding violence or personal application of violent means, for illegitimate personal gain, or negligence in its traditional meaning. I shall present some of these cases and focus on the novelty of the criminal prosecution of purely abstract symbol processing work. I also address the special place of the judicial system in the division of power when neither the legislative nor the executive system seem geared to control these activities. I close with my recommendation of a code of conduct for mathematicians working in a consulting function for governments or companies. This is joint work with Assoc. Prof. of Law Kristian Cedervall Lauta of Copenhagen University

Jochen Brüning (Humboldt)

Title: Some considerations on controlled stratified topological spaces

Abstract: The axioms for smoothly stratified spaces as formulated by John Mather are fairly complicated and lack motivation. As a consequence, there are many variations around that make comparison and often comprehension quite difficult. In the talk I will outline a (hopefully) rather natural explanation of the axioms on the topological side alone.

Igor Burban (Cologne)

Title: Algebraic geometry of the classical Yang-Baxter equation

Abstract: In my talk, I am going to explain a geometric description of solutions of the classical Yang-Baxter equation (CYBE). Namely, starting with any pair (E, A), where E is an irreducible plane cubic curve and A a torsion free sheaf of Lie algebras (whose generic fiber is a given complex simple Lie algebra) with vanishing cohomology, one can associate to it in a canonical way a solution of CYBE. It turns out that at least all elliptic and rational solutions of CYBE arise in this way. The developed method will be illustrated by explicit examples. This talk is based on joint works with Lennart Galinat, Thilo Henrich and Alexander Stolin.

Alberto Cattaneo (Zürich)

Title: An introduction to the BV-BFV formalism

Abstract: The BV-BFV formalism unifies the BV formalism (which deals with the problem of fixing the gauge of field theories on closed manifolds) with the BFV formalism (which yields a cohomological resolution of the reduced phase space of a classical field theory). I will explain how this formalism arises and how it can be quantized.

Ksenia Fedosova (Freiburg)

Title: Eisenstein series twisted by representations with non-expanding cusp monodromy

Abstract: In this talk, we investigate the behavior of the Eisenstein series, or generalized eigenfunctions of the Laplace operator on hyperbolic surfaces. We twist them by a (possibly) non-unitary representation of the fundamental group of the manifold, show their convergence on some half-plane and study their Fourier expansion.

Batu Güneysu (Humboldt)

Title: Covariant Riesz transforms and the Calderon-Zygmund inequality

Abstract: I explain how one can use a new probabilistic Bismut formula for the covariant derivative of the heat semigroup acting on 1-forms on a noncompact Riemannian manifold M, in order to prove the L^p boundedness of the induced covariant Riesz transform. The main strength of this result stems from the fact that no injectivity radius assumptions on M are made, only boundedness assumptions on the curvature tensor. As a consequence of this result, we obtain several new L^p results for geometric analysis, such as an L^p Calderon-Zygmund inequality, L^p estimates for Solutions of the Poisson equation, or precompactness results for sequencence of isometric immersions. This is joint work with R. Baumgarth and S. Pigola.

Klaus Kröncke (Hamburg)

Title: Stability of Ricci de Turck flow on Singular Spaces

We establish stability of the Ricci de Turck flow near Ricci-flat metrics with isolated conical singularities. More precisely, we construct a Ricci de Turck flow which starts sufficiently close to a Ricci-flat metric with isolated conical singularities and converges to

a singular Ricci-flat metric under an assumption of integrability, linear and tangential stability. We provide a characterization of conical singularities satisfying tangential stability and discuss examples where the integrability condition is satisfied. This is joint work with Boris Vertman.

Semyon Klevtsov (Cologne)

Title: Geometry and large N asymptotics in Laughlin states

Laughlin states are N-particle wave functions, which successfully describe fractional quantum Hall effect (QHE) for plateaux with simple fractions. It was understood early on, that much can be learned about QHE when Laughlin states are considered on a Riemann surface. I will define the Laughlin states on a compact oriented Riemann surface of arbitrary genus and talk about recent progress in understanding their geometric properties and relation to physics. Mathematically, it is interesting to know how do Laughlin states depend on an arbitrary Riemannian metric, magnetic potential function, complex structure moduli, singularities -- for a large number of particles N. I will review the results, conjectures and further questions in this area, and relation to topics such as Coulomb gases/beta-ensembles, Bergman kernels for holomorphic line bundles, Quillen metric, zeta determinants.

Matthias Lesch (Bonn)

Title: On the domain of the Gauss Bonnet operator on stratified spaces

Abstract: We consider a generalized Dirac operator on a compact stratified space with an iterated cone-edge metric. Assuming a spectral Witt condition, we prove its essential self-adjointness and identify its domain and the domain of its square with weighted edge Sobolev spaces. This sharpens previous results where the minimal domain is shown only to be a subset of an intersection of weighted edge Sobolev spaces. Our argument does not rely on microlocal techniques and is very explicit. The novelty of our approach is the use of an abstract functional analytic notion of interpolation scales. Our results hold for the Gauss-Bonnet and spin Dirac operators satisfying a spectral Witt condition. This is a joint work with Luiz Hartmann and Boris Vertman.

Xiaonan Ma (Paris)

Title: Generalized Bergman kernels on symplectic manifolds and applications.

Abstract: A suitable notion of ``holomorphic section'' of a prequantum line bundle on a compact symplectic manifold is the eigensections of low energy of the Bochner Laplacian acting on high p-tensor powers of the prequantum line bundle. We explain

the asymptotic expansion of the corresponding kernel of the orthogonal projection as the power p tends to infinity. This implies the compact symplectic manifold can be embedded in the corresponding projective space. With extra effort, we show the Fubini-Study metrics induced by these embeddings converge at speed rate $1/p^{2}$ to the symplectic form. We explain also its implication on Bezerin-Toeplitz quantizations.

Niels Martin Møller (Copenhagen)

Title: Non-Compactness of Moduli Spaces of Minimal Surfaces via PDE-Gluing

Abstract: We consider moduli spaces M(g,r) of complete embedded minimal 2-surfaces (in Euclidean 3-space) of final total curvature with genus g and r ends. It is expected (Hoffman-Meeks & Ros) that M(g,r) must be empty when r > g + 2, and also that (for r > 3) the moduli spaces are non-compact whenever non-empty. Families exhibiting such non-compactness come about by considering the gluing of catenoids as done by Kapouleas in 1997, but now allowing the intersection angles to degenerate to zero, which can be arranged to happen uniformly in the genus. I will discuss the most basic case of this program, r = 4 and g = 1, which viewed at appropriate scales is really a doubling construction for the flat plane, and where the new examples are only Alexandrov embedded. This work is joint with Stephen Kleene (U Rochester).

Werner Müller (Bonn)

Title: Analytic torsion of hyperbolic manifolds with cusps

Abstract: I will report on joint work with F. Rochon. We consider a hyperbolic manifold with cusps and certain flat unimodular bundles over this manifold. We use renormalized traces in the sense of Melrose to define the regularized anayltic torsion for this set up. The main result is a formula which relates the regularized analytic torsion to the Reidemeister torsion of the natural compactification of the hyperbolic manifold. Our method relies on the study of the behavior of the spectrum of the Hodge Laplacian on a closed manifold undergoing degeneration to a manifold with cusps. At the end I will discuss some applications.

Holger Bech Nielsen (Copenhagen)

Title: Deriving effective locality from diffeomorphism symmetry'

As a part of the project on which we long worked of deriving at the end all the laws of nature from a random very complicated mathematical structure, a project we called for long ``Random dynamics'' we want specifically to derive the principle of locality from diffeomorphism symmetry. We assume a very general - and non-local - action for a world in which there is a manifold with (local) fields defined on it, but we do NOT assume the action to be local at first. However, we assume it be in a functional way Taylor

expandable. That is to say we generalize Taylor expansion to functionals. As a result we get an in practice local theory. However, the coupling constants ``know'' about far away and future space time points.

William Elbæk Petersen (Aarhus)

Title: Asymptotic expansions of Witten-Reshetikhin-Turaev invariants

Abstract: The level k Witten-Reshetikhin-Turaev (WRT) invariant of a closed oriented three manifold containing a link was introduced by Witten in order to give an intrinsic interpretation of the Jones polynomial associated to a link, in terms of quantum Chern-Simons theory which is a gauge theory. In this picture, the Jones polynomial evaluated at a k-th tooth of unity emerge as the level k expectation value of certain observables. Moreover; Witten proposed that these invariants extends to topological quantum field theories (TQFT's) in a way that involves geometric quantization of moduli spaces of flat connections. Though Witten's argument relied on Feynman path-integrals, TQFT's were subsequently constructed rigorously from a mathematical point of view through combinatorial means by Reshetikhin and Turaev, and it is now known that these TQFT's can also be realized using quantization of moduli spaces. From the path-integral motivation, it is conjectured that the WRT-invariants admits asymptotic expansions in the level k. In this talk, I shall present recent results on this conjecture joint with my supervisor Prof. J.E. Andersen. Our approach is based on quantization of moduli spaces, and our results are obtained in the more general context of quantization of Kähler manifolds.

Paolo Piazza (Rome, Sapienza)

Title: Singular spaces, metrics of positive scalar curvature and K-theory invariants of the spin Dirac operator.

Abstract: Let $\widehat{X}$ be a stratified pseudomanifold and let X be its resolution, a manifold with fibered corners. We endow X with a fibered-corner metric (also known as a Φ-metric). We assume that the strata of $\widehat{X}$ are spin. Under a positive scalar curvature assumption along the links we define and study the fundamental class and the index class of the associated spin Dirac operator. If the metric has positive scalar curvature everywhere then we define the rho class and give its main properties. We illustrate how groupoid techniques can be used in this context and how they give results on more singular structures, such as singular foliations. This is joint work with Vito Felice Zenobi.

Julie Rowlett (Gothenburg)

Title: One can hear the corners of a drum

Abstract: Analytically computing the spectrum of the Laplacian is impossible for all but a handful of classical examples. Consequently, it can be tricky business to determine which geometric features are spectrally determined; such features are known as geometric spectral invariants. Weyl demonstrated in 1912 that the area of a planar domain is a geometric spectral invariant. In the 1950s, Pleijel proved that the perimeter is also a spectral invariant. Kac, and McKean & Singer independently proved in the 1960s that the Euler characteristic is a geometric spectral invariant for smoothly bounded domains. At the same time, Kac popularized the isospectral problem for planar domains in his article, ``Can one hear the shape of a drum?" Colloquially, one says that one can ``hear" spectral invariants. Hence the title of this talk in which we will show that the presence, or lack, of corners is spectrally determined. In the process, we will see how a certain ``corner contribution" to the heat trace is obtained by explicitly calculating the Green's functions for infinite sectors with Dirichlet, Neumann, Robin, and mixed boundary conditions. Moreover, using microlocal techniques, we will see that this corner contribution is universal. Finally, we will show how the results generalize to surfaces. This talk is based on current joint work with M. Nursultanov and D. Sher, and previous joint work with Z. Lu.

Nikhil Savale (Cologne)

Title: The Bergman kernel expansion for semipositive line bundles on a Riemann surface

Abstract: We generalize the results of Montgomery for the Bochner Laplacian on high tensor powers of a line bundle. When specialized to Riemann surfaces, this leads to the Bergman kernel expansion for semi-positive line bundles whose curvature vanishes at finite order. A novel feature of the expansion, at points where the curvature vanishes, is the presence of fractional exponents given in terms of the order of vanishing. Based on joint work with George Marinescu.

Elmar Schrohe (Hannover)

Title: Index Theory for Operators Associated with Groups of Quantized Canonical Transformations

Abstract: Given a Lie group G of quantized canonical transformations acting on the space $L^2(M)$ over a closed manifold M, we define an algebra of so-called G-operators on $L^2(M)$. We show that to G-operators we can associate symbols in appropriate crossed products with G, introduce a notion of ellipticity and prove the Fredholm property for elliptic elements. This framework encompasses many known elliptic theories, for instance, shift operators associated with group actions on M, transversal elliptic theory,

transversally elliptic pseudodifferential operators on foliations, and Fourier integral operators associated with coisotropic submanifolds. We also point out how the index can be computed in case G is a discrete group.
(Joint work with Anton Savin and Boris Sternin)

Thomas Ostergaard Sorensen (München, LMU)

Title: Pointwise estimates on derivatives of Coulombic wave functions and their electron densities

Abstract: In this talk we discuss (optimal) a priori pointwise bounds for all derivatives of non-relativistic Coulombic eigenfunctions psi, involving negative powers of the distance to the singularities of the many-body potential. If time permits, we discuss consequences for the corresponding electron density. This is joint work (arXiv:1803.03495) with S. Fournais (Aarhus, DK)

Alexander Strohmeier (Leeds)

Title: Index Theory of the Dirac Operator on Spacetimes

Abstract: I will set the natural context in which the Dirac operator on a curved spacetime with certain boundary conditions becomes a Fredholm operator. The index of this operator can be computed by an analogue of the Atiyah-Patodi-Singer index formula. I will discuss a local version of this theorem and its relation to computations in the physics literature in the context of the chiral anomaly. (Joint work with C. Baer)

Niels Waterstraat (Kent)

Title: On the Fredholm Lagrangian Grassmannian, Spectral Flow and ODEs in Hilbert Spaces

We consider homoclinic solutions for Hamiltonian systems in symplectic Hilbert spaces and generalise spectral flow formulas that were proved by Pejsachowicz and the author in finite dimensions some years ago. Roughly speaking, our main theorem relates the spectra of infinite dimensional Hamiltonian systems under homoclinic boundary conditions to intersections of their stable and unstable spaces. Our proof has some interest in its own. Firstly, we extend a celebrated theorem by Cappell, Lee and Miller about the classical Maslov index in $\mathbb{R}^{2n}$ to symplectic Hilbert spaces. Secondly, we generalise the classical index bundle for families of Fredholm operators of Atiyah and Jänich to unbounded operators for applying it to Hamiltonian systems under varying boundary conditions. Finally, we make use of striking results by Abbondandolo and Majer to study Fredholm properties of infinite dimensional Hamiltonian systems.

Steven Morris Zelditch (Northwestern)

Title: Interface asymptotics of Bergman kernels

Abstract: Let (M, L, ω) be a polarized Kähler manifold (i.e. $L \to M$ is an ample line bundle). Let $D \subset M$ be a domain. How do you ``fill up'' D with quantum states that do not `spill over'' into M - D? A quantum state is a holomorphic section of $L^k$. This question was raised by physicists in the quantum Hall effect and has many possible interpretations. In a series of articles, Peng Zhou and I have been studying it via ``partial Bergman kernels', i.e. projections onto subspaces of holomorphic sections. The subspace is adapted to the domain using spectral theory of Berezin-Toeplitz operators. The partial density of states is basically 1 on D, almost zero on M - D and has a Gaussian interface along the boundary of D. This gives a kind of CLT (central limit theorem) in Kahler geometry. We also prove a CLT in the setting of toric Kaehler manifolds for a family of discrete measures living on the moment polytope.

Chaofeng Zhu (Nankai)

Title: The Maslov Index in Symplectic Banach Spaces (joint with Bernhelm Booss-Bavnbek, published Mem. AMS.)

Abstract: We consider a curve of Fredholm pairs of Lagrangian subspaces in a fixed Banach space with continuously varying weak symplectic structures. Assuming vanishing index, we obtain intrinsically a continuously varying splitting of the total Banach space into pairs of symplectic subspaces. Using such decompositions we define the Maslov index of the curve by symplectic reduction to the classical finite-dimensional case. We prove the transitivity of repeated symplectic reductions and obtain the invariance of the Maslov index under symplectic reduction, while recovering all the standard properties of the Maslov index. As an application, we consider curves of elliptic operators which have varying principal symbol, varying maximal domain and are not necessarily of Dirac type. For this class of operator curves, we derive a desuspension spectral flow formula for varying well-posed boundary conditions on manifolds with boundary and obtain the splitting formula of the spectral flow on partitioned manifolds.