Schedule Minicourses 7-9 September

Wednesday

10.00 – 12:00	Eric Carlen
14:00	Coffee
14:30-16:30	Jean Dolbeault

Thursday

10:00-11:00	Eric Carlen
11:00-12:00	Jean Dolbeault

Friday

10.00 – 12:00	Jean Dolbeault
14:00	Coffee
14:30-16:30	Eric Carlen

Speakers

Eric Carlen (Rutgers Univ)

Functional Inequalities and Gradient Flow for Quantum Evolution Equations

In recent years, building on work of Felix Otto, much progress has been made in the study of a wide class of evolution equations for probability densities by viewing them as gradient flow for certain entropy functions with respect to mass-transportation metrics.

The simplest example is the classical Fokker-Planck equation, which was shown by Jordan, Kinderleher and Otto to be gradient flow in the 2-Wasserstein metric for the relative entropy with respect to the steady-state Gaussian density. The Fokker-Planck equation has several natural quantum analogs, in particular one for fermions. This has the form of a Lindblad evolution equation for a time-dependent density matrix. There is a natural differential structure that allows this equation to be written as a "non-commutative partial differential equation", and also, as was shown by myself and Jan Mass, to define a natural analog of the 2-Wasserstein distance as a Riemannian distance on the manifold of density matrices such that the equation is, as in the classical case, gradient flow in this metric for the relative entropy with respect to the ground state.

Recent joint work with Jan Maas has extended this to a wide class of quantum evolutions equations, linear and non-linear. As in the classical case, a wide range of functional inequalities governing the evolution can be seen as consequences of convexity with respect to the underlying transport metric. These lectures will provide an introduction form the beginning to quantum evolution equations and gradient flow. The close parallels with the classical theory will be emphasized, and a number of open problems will be pointed out.

Jean Dolbeault (Université Paris-Dauphine)

Symmetry and nonlinear diffusions

The goal of the minicourse is to sketch the proof of symmetry results in weighted interpolation inequalities which have been obtained recently in a joint work with Maria J. Esteban and Michael Loss in the critical case, and extended in another paper with Maria J. Esteban, Michael Loss and Matteo Muratori in the subcritical case. The strategy relies on entropy methods and non-linear diffusion equations. The course will be organized as follows:

1) Entropy methods for nonlinear diffusions: self-similar variables and Rényi entropies

2) Flows on compact manifolds

3) Symmetry and symmetry breaking in Caffarelli-Kohn-Nirenberg inequalities