

SCHEDULE

Summer School at Institute Mittag Leffler

TWO APPROACHES TOWARDS LOCAL UNIFORMIZATION AND RESOLUTION OF SINGULARITIES IN CHARACTERISTIC $p > 0$.

MAY 23 - 27, 2016

Breakfast is served between 07:30-09:00 Mon - Fri in Gula Villan (the Yellow Villa).

	Monday	Tuesday	Wednesday	Thursday	Friday
08:55 - 09:10	<i>Weclome</i>				
09:10 - 10:10	Piltant I	Cossart I	Teissier II	Teissier III	Teissier V
10:10 - 10:20	<i>short break</i>	<i>short break</i>	<i>short break</i>	<i>short break</i>	<i>short break</i>
10:20 - 11:20	Cutkosky	Benito	Piltant II	Cossart II	Piltant IV
11:20 - 11:50	<i>tea & coffee</i>	<i>tea & coffee</i>	<i>tea & coffee</i>	<i>tea & coffee</i>	<i>tea & coffee</i>
11:50 - 12:50	Kuhlmann	Spivakovsky	Jannsen I	Piltant III	Cossart III
12:50 - 14:30	<i>lunch</i>	<i>lunch</i>	<i>lunch</i>	<i>lunch</i>	<i>lunch</i>
14:30 - 15:30	Temkin	Teissier I		Teissier IV	
15:30 - 16:00	<i>tea & coffee</i>	<i>tea & coffee</i>		<i>tea & coffee</i>	
16:00 - 17:00	Tevelev I	Tevelev II		Jannsen II	
17:00 - 17:30					

The conference dinner will take place on Thursday evening at 19h00.

Here is a list of the talks in chronological order with some more information:

————— **Monday** —————

0. Welcome: Some organizational informations.

1. Olivier Piltant: Valuations and local uniformization.

Abstract: This is an introductory talk about valuations and their use in resolution of singularities.

2. Dale Cutkosky: Graded rings associated to valuations and the condition of finite generation.

Abstract: Suppose that K is a valued field which dominates a noetherian local ring R with quotient field R . We discuss some graded rings associated to the valuation on R which provide fundamental information about birational properties of the ring. We consider especially differences between characteristic zero and positive characteristic and the question of finite generation or relative finite generation under a finite extension.

3. Franz Viktor Kuhlmann: Ramification theory, elimination of ramification, and the defect.

Abstract: In my talk I will present:

- 1) an introduction to the basic definitions and facts from ramification theory,
- 2) the notions of tame and wild ramification,
- 3) the task of elimination of ramification in valued function fields and its meaning for local uniformization and for embedding lemmas which are used in the model theory of valued fields,
- 4) the notion of defect and a couple of examples of finite extensions of valued fields with nontrivial defect,
- 5) the way in which ramification theory is used to reduce the study of wild ramification and the defect to the study of normal extensions of degree p (which over fields of characteristic p are either purely inseparable or Artin-Schreier extensions),
- 6) an example showing why the defect is an obstacle to the elimination of ramification.

4. Michael Temkin: On recent advances in the theory of valued fields and some applications.

Abstract: I will talk about some recent advances in the theory of extensions of valued fields and recent applications to the following areas:

- 1) Inseparable local uniformization.
- 2) Resolution of varieties by p -alterations.
- 3) Wild covers of Berkovich curves (parts are joint with Cohen and Trushin).
- 4) Automorphisms of completed algebraic closure of $k((t))$ and tilting equivalents of C_p (joint with Kedlaya).

5. Jenia Tevelev: Tropical compactifications and applications to Teissier's question I.

Abstract : We showed in [T2] that any embedded resolution of singularities is induced by an equivariant map of toric varieties, thus providing an affirmative answer to a question of Teissier. The proof is based on the method of tropical compactifications [T1,HKT] and in fact is simply a souped-up version of the theorem from [LQ] (conjectured by the author). The goal of the lectures is to explain these ideas and to sketch the proof.

[T1] J. Tevelev, Compactifications of Subvarieties of Tori, Amer. J. Math, 129, no. 4 (2007)

[HKT] P. Hacking, S. Keel, and J. Tevelev, Stable pair, tropical, and log canonical compact moduli of del Pezzo surfaces, Inventiones, 178, no. 1 (2009)

[LQ] M. Luxton; Z. Qu, Some results on tropical compactifications, Trans. Amer. Math. Soc., 363 (2011)

[T2] J. Tevelev, On a Question of Teissier, Collectanea Math., 65, no. 1 (2014)

————— **Tuesday** —————

6. Vincent Cossart: Introduction, statement of the Theorem, strategy of proof.

Summary: reduction to Local uniformization, valuation of rational rank 1, algebraic residual extension. Hironaka's invariants.

7. Angélica Benito: Desingularization in dimension two.

Abstract: In this talk we will discuss Villamayor (and co) approach to the problem of resolution of singularities in positive characteristic. In this approach the idea of restriction to a hypersurface of maximal contact is replaced by the notion of elimination (of variables) using Rees Algebras.

We will review some of the results that we have obtained in the last years: definition of some invariants, classification of some nice cases where resolution can be achieved and, finally, we will sketch some ideas of the proof of resolution of singularities for 2-dimensional schemes. Time permitting, we will also discuss some pathologies and some new invariants of the case of positive characteristic we have defined recently.

8. Mark Spivakovsky: Extension of valuations centered in a local domain to its completion.

Abstract: Let (R, m, k) be a local noetherian domain with field of fractions K , let

$$\nu : K^* \rightarrow \Gamma$$

be a valuation centered at R and let R_ν be the corresponding valuation ring of K , dominating R . Denote by \hat{R} the m -adic completion of R . In the applications of valuation theory to commutative algebra and the study of singularities, one is often induced to replace R by its m -adic completion \hat{R} and ν by a suitable extension $\hat{\nu}$ to $\frac{\hat{R}}{P}$ for a suitably chosen prime ideal P , such that $P \cap R = (0)$. In [1] we gave a systematic description of all such extensions $\hat{\nu}$ and defined the notion of **tight** extensions, which are of particular interest for applications. One of the key properties of a tight extension $\hat{\nu}$ is that its value group equals Γ and its graded algebra is birational to that of ν . The existence of such extensions (known as Teissier's conjecture) is a crucial step in at least two recent approaches to local uniformization in positive characteristic. While our work on Teissier's conjecture in its full generality is still in progress, in this talk we will explicitly describe tight extensions in two special cases: the case when $\text{rk } \nu = 1$ and the case when ν is Abhyankar, that is, when Abhyankar's inequality for ν is, in fact, an equality.

References 1. J. Herrera, M. A. Olalla, M. Spivakovsky, B. Teissier, Extending a valuation centered in a local domain to its formal completion, Proceedings of the London Mathematical Society (3) 105 (2012), 571621.

9. Bernard Teissier: Outline of the proof and embedded resolution of affine toric varieties.

Abstract: I will explain the general strategy of reduction of local uniformization to the embedded resolution of affine toric varieties, by specialization to the associated graded ring of the valuation. Then I will recall the result, joint with Pedro González Pérez, of embedded resolution of affine toric varieties.

10. Jenia Tevelev: Tropical compactifications and applications to Teissier's question II. See the abstract of the first talk by Tevelev.

————— Wednesday —————

11. Bernard Teissier: Overweight deformations and the local uniformization of the associated valuations.

Abstract: I will define overweight deformations of weighted affine toric varieties, explain how they produce valuations and how these valuations can be uniformized by some of the embedded resolutions of the affine toric variety.

12. Olivier Piltant: Local uniformization reduces to p -cyclic extensions.

Summary: reduction to the complete local case, reduction to the hypothesis (\mathbf{G}) , from valuation rings to local models.

13. Uwe Jannsen: Directrix, Ridge and Hironaka schemes I.

Abstract: I explain some basic invariants important for resolution, explain the value of the ridge

and the directrix for finding the near points, and for giving invariants. I will also report on work of my student Bernhard Dietel on Hironaka schemes.

Thursday

14. **Bernard Teissier: The valuative Cohen theorem.**

Abstract: I will explain the proof of the theorem which produces equations for the toric degeneration of a complete equicharacteristic noetherian domain to the associated graded ring of a rational valuation, so that every such valuation is obtained by overweight deformation from a weighted affine toric variety, provided that its semigroup of values is finitely generated. The reduction of the case of excellent equicharacteristic local domains to the complete case will be treated if time permits.

15. **Vincent Cossart: Characteristic polyhedra, definition of the invariant ω under the hypothesis (G), permissible blowing ups.**

Summary: Hironaka's polyhedron leads to monomial valuations. Applying differential operators to initial forms of the equation, we define the invariant ω . We solve the case $\omega(x) = 0$.

16. **Olivier Piltant: Structure of initial forms in p -cyclic extensions.**

Summary: initial forms w.r.t. compact faces of the characteristic polyhedron are Artin-Schreier or purely inseparable. They belong to polynomial rings over the residue field (of positive characteristic).

17. **Bernard Teissier: The semigroups of values of rational Abhyankar valuations is quasi-finitely generated.**

Abstract: I will explain how by a birational map an Abhyankar rational valuation on a complete equicharacteristic local domain can be transformed into a rational valuation with finitely generated semigroup of values.

18. **Uwe Jannsen: Directrix, Ridge and Hironaka schemes II.**

See the abstract of the first talk by Jannsen.

Friday

19. **Bernard Teissier: Approaching the general case.**

Abstract: I will explain how a rational valuation of rational rank r on the complete local domain R can be approximated arbitrarily by Abhyankar semivaluations on r -dimensional quotients of R and how some local uniformizations of these semivaluations can probably uniformize the original valuation provided they approximate it well enough.

20. **Olivier Piltant: Three properties of the function $(mult, \omega)$.**

Summary: refining the multiplicity by ω produces a constructible function, stable under regular base change. The stratum $mult = p, \omega \geq 1$ is closed of codimension at least two.

21. **Vincent Cossart: Definition of κ , zoology.**

Summary: we define the invariant $\kappa(x) \in \{1, 2, 3, 4\}$. We solve the case $\kappa(x) = 1$ and give some hints for the cases $\kappa(x) = 2$ or 3 .