Mittag-Leffler Institute

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New directions in mathematics of Coulomb gases and quantum Hall effect

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Jørgen Andersen *The Hitchin connection in Kähler quantisation*
Abstract : In this talk we will first review the general setting of Hitchin connections in the context of geometric Kähler quantisation. We will then discuss our initial generalisation of the construction of the Hitchin connection for rigid families of complex structures. We will then go on to discuss our more recent work on obstructions and constructions of Hitchin type connections given by higher order differential operators for general families of Kähler structures on symplectic manifolds including our construction of formal Hitchin connections relevant in deformation quantisation. Towards the end of the talk we will consider generalisation to case of Hitchin type connections, which are given by infinite order differential operators.

Tankut Can *Coulomb Gas in Superspace*
Abstract : The Pfaffian quantum Hall wave function can be represented by a Laughlin wave function on a super Riemann surface. This leads to a plasma mapping of the Pfaffian state to a one-component Coulomb plasma in superspace. Motivated by this correspondence, I will discuss the statistical mechanics of Coulomb plasmas on a super Riemann surface. Special emphasis will be placed on Ward identities (loop equations) used to solve for correlation functions. Finally, I will comment on the consequences for observables in the Pfaffian state, and highlight novel correlation functions which come to light in superspace.

Laurent Charles *From Weyl law to area law for entanglement entropy*
Abstract : The Weyl law gives the mean distribution of eigenvalues of an operator in terms of its symbol. I will discuss the case of discontinuous symbols, for truncated Toeplitz matrices and for Berezin-Toeplitz operators. I will then present some applications to complex random matrices : typically, in the Ginibre ensemble, we estimate the variance for the number of eigenvalues in a smooth domain. Another application is a mathematical proof of the area law for the entanglement entropy for fermions in quantum Hall effect.

Benoit Estienne *Entanglement entropy and Full Counting Statistics in the Integer Quantum Hall effect*
Abstract : Ideas coming from quantum information theory have provided invaluable insights and powerful tools for quantum many-body systems. One of the most basic tools in the arsenal of quantum information theory is entanglement entropy. A particularly striking phenomenon is the area law of the entanglement entropy, which has been widely discussed in recent years in condensed matter and quantum field theories. Typically, one considers a many-particle state and a geometric partition of the space in two sub-regions. The von Neumann entropy of the reduced state of a sub-region measures the degree of entanglement between the two regions. The area law states that the leading semiclassical asymptotic of the entanglement entropy is proportional to the volume of the boundary of the sub-region.

I will start with an introduction to quantum entanglement, entanglement entropy and their applications in condensed matter. I will then present some recent results obtained with L.Charles in the context of the quantum Hall effect regarding both the entanglement entropy and full counting statistics : proof of the Area law (arXiv :1803.03149) as well as unpublished results regarding the first subleading corrections in the bulk, and critical behavior coming from the massless edge modes.

Andrey Gromov *Introduction to the Quantum Hall effect*
Abstract : Introductory lecture on the Quantum Hall effect.

Andrey Gromov *Collective modes in fractional QHE phases*
Abstract : I will discuss neutral collective excitations in fractional quantum Hall (FQH) phases. To start, I will review the Girvin-MacDonald-Platzman mode, which is a gapped mode generated by an area-preserving distortion (APD) of the quantum Hall liquid. All known FQH states support this excitation. For example, Laughlin state supports only this excitation. Next I will turn to the Moore-Read pfaffian state, which supports two neutral modes – one boson and one fermion. Bosonic mode is still described by an APD, while the fermionic mode is not. The main objective of the talk is to develop an approach to the Moore-Read state that allows to treat the two modes with analytically. The main idea is to place the Moore-Read state on a superspace, where all equations become simple. In a certain sense these two collective modes are superpartners.

Alice Guionnet *Large deviations in Random Matrix Theory*
Abstract : Introductory lecture on Large deviations in Random Matrix Theory.

Adrien Hardy *A CLT for the circular beta-ensemble at high temperature*
Abstract : We consider the circular beta-ensemble when the inverse temperature beta is of order $1/N$ and describe the fluctuations of this particle system around its equilibrium measure. The limiting fluctuations turns out to be Gaussian field with a covariance structure that interpolates between the L^2 norm and $H^{1/2}$ norm structure. We also discuss the more general setting where we add a potential V to the particle system ; in this case the limiting covariance is not universal and depends on V in a semi-explicit way.

Håkan Hedenmalm *Planar orthogonal polynomials and boundary universality for random normal matrices*
Abstract : We obtain an asymptotic expansion for the orthogonal polynomials with respect to exponentially varying weights. This formula is applied to yield error function universality for the interface at regular boundary points. This reports on joint work with A. Wennman.

Nam-Gyu Kang *Calculus of conformal fields on a compact Riemann surface*
Abstract : I will present analytical implementation of conformal field theory on a compact Riemann surface. After explaining Ward identities which represent the Lie derivative operators in terms of the Virasoro fields and the puncture operators associated with the background charges, I will derive Eguchi-Ooguri's version of Ward's equations on a torus and give physical interpretations of various formulas in function theory e.g. analogues of addition theorems. If time permits, I will explain boson-fermion correspondence in the genus zero and one cases with a mathematical rigor. Joint work with Nikolai Makarov.

Anton Kapustin *Thermal Hall conductance as a relative topological invariant, or detecting gravitational anomalies for lattice systems*
Abstract : It is well-known that zero-temperature Hall conductance of a 2d system can be interpreted both as a bulk transport coefficient and a U(1) anomaly for the edge modes. The

former interpretation allows one to write down a simple formula for it (Kubo formula). The latter interpretation explains why Hall conductance is a topological invariant. In this talk I will explain the difficulties in extending these considerations to thermal Hall conductance and how they are overcome. I argue that thermal Hall conductance should be regarded as an exact 1-form on the parameter space rather than a function. Another observation is that anomalies of edge modes can be interpreted in physical terms as violations of F. Bloch ?s theorem about the absence of currents in an equilibrium state. Combining these observations, I show that low-temperature thermal Hall conductance of a gapped 2d system is robust under arbitrary deformations which do not close the gap.

Alisa Knizel *Asymptotics of discrete β-corners processes via two-level discrete loop equations*
Abstract : We introduce and study stochastic particle ensembles which are natural discretizations of general β-corners processes. We prove that under technical assumptions on a general analytic potential the global fluctuations for the difference between two adjacent levels are asymptotically Gaussian. The covariance is universal and remarkably differs from its counterpart in random matrix theory. Our main tools are certain novel algebraic identities that are two-level analogues of the discrete loop equations. Based on joint work with Evgeni Dimitrov (Columbia University).

Thomas Leblé *Fluctuations of linear statistics in two-dimensional Coulomb gases*
Abstract : I will present results concerning the fluctuations of two-dimensional gases of classical particles with Coulomb interaction ("one-component plasma"), at arbitrary temperature. Combining Johansson's trick with a transportation scheme and a fine understanding of the energy, we prove a CLT for fluctuations of test functions with a few degrees of regularity. This can be rephrased in terms of convergence to a Gaussian Free Field. I will also mention open problems beyond the smooth case.

Xiaonan Ma *Determinant line bundle and Quantum Hall effect*
Abstract : We study the generating functional, the adiabatic curvature and the adiabatic phase for the integer quantum Hall effect (QHE) on a compact Riemann surface. For the generating functional we derive its asymptotic expansion for the large flux of the magnetic field, i.e., for the large degree k of the positive Hermitian line bundle L^k. We observe the relation between the Bismut-Gillet-Soulé curvature formula for the Quillen metric and the adiabatic curvature for the electromagnetic and geometric adiabatic transport of the integer Quantum Hall state.

Mylène Maïda *A statistical physics approach to the sine beta process*
Abstract : The Sine beta processes are a family of point processes introduced by Valko and Virag and shown to be the bulk limit of Gaussian beta ensembles in random matrix theory. They have been defined through a one-parameter family of SDEs coupled by a two-dimensional Brownian motion (or more recently as the spectrum of a random operator). Through these descriptions, some properties have been derived by Holcomb, Paquette, Valko, Virag and others but there is still much to understand. In a work with David Dereudre, Adrien Hardy (Université de Lille) and Thomas Leblé (Courant Institute, New York), we use tools from classical statistical mechanics based on DLR equations to give a completely different description of the Sine beta process and derive some properties, such as rigidity and tolerance.

Nicolas Regnault *Living on the edge : Model states for the interface of chiral topological* *orders*

Abstract : Interfaces between topologically distinct phases of matter reveal a remarkably rich phenomenology. To go beyond effective field theories, we study two prototypical examples of such an interface : Between two Abelian states (the Laughlin and Halperin states) or between an abelian and a non abelian states. Using matrix product states, we propose a family of model wavefunctions for the whole system including both bulks and the interface. We show through extensive numerical studies that it unveils both the universal properties of the system, such as the central charge of the gapless interface mode and its microscopic features. It also captures the low energy physics of experimentally relevant Hamiltonians. For the abelian/non-abelian case, we show that a Majorana mode is trapped at the interface. Our approach can be generalized to other phases described by tensor networks.

Rémi Rhodes *Sine-Gordon model and neutral log-gases*

Abstract : In this talk, I will discuss the Sine-Gordon model and its relation with neutral log-gas. This is a model of electric charges with positive/negative signs interacting through a logarithmic potential (or Coulomb potential in 2D). In the case when particles are confined to the boundary of a 2D domain (called boundary Sine-Gordon model) I will present a short proof that allows us to completely solve the ultraviolet renormalization of this model until the Berezinky-Kosterlitz-Thouless (BKT) transition, including renormalization of correlation functions. The method is purely probabilistic and relies on concentration methods for martingales.

Nicolas Rougerie *Perturbing the Laughlin state*

Abstract : An important part of Laughlin's famous theory of the fractional quantum Hall effect is that his proposed wave-function is an exact ground state for a toy hamiltonian with short range interactions and translation invariance.

An equally important part of the theory is that the proposed wave-function should be robust against various forms of perturbations : lowering the filling factor slightly, including trapping/impurity potentials, taking the long range part of the interaction into account ...

I shall discuss mathematical results motivated by these expectations. The problem at hand is to minimize the perturbations within the (large !) class of functions that exactly minimize the toy hamiltonian. Our main result is that it is sufficient (in the large particle number limit) to generate uncorrelated quasi-holes on top of Laughlin's wave function.

Based on joint works with Elliott Lieb, Alessandro Olgiati and Jakob Yngvason.

Sylvia Serfaty *Introduction to Coulomb gases*

Abstract : Introductory lecture on Coulomb gases.

Paul Wiegmann *Razor-edge boundary of Coulomb gas*

Abstract : TBA

Wei Wu *Spin models, lattice gases and homogenization*

Abstract : Lattice gas representation is a useful approach to study spin models with Abelian symmetry (e.g., the XY and the Villain model). Using the lattice gas representation, Frohlich and Spencer rigorously established the existence of Kosterlitz-Thouless transition in the XY and the Villain model in d=2 ; and the existence of long range order at low temperature in d>=3. We will briefly revisit their construction, and report on some recent working progress (based on joint works with Scott Armstrong and Paul Dario) with the additional input of quantitative homogenization.

Jakob Yngvason *Rigidity of the Laughlin Liquid*

Abstract : To explain the so-called fractional quantum Hall effect for electrons moving in a strong magnetic field, R.B. Laughlin suggested in 1983 a many-body wave function that bears his name. A crucial property of this function, supported by numerical and experimental evidence, is its extreme rigidity with respect to perturbations by impurities or external fields. In the talk one aspect of this rigidity will be discussed : modifications of the wave function that retain the correlations present in the Laughlin state cannot compress the one-particle density (suitably averaged) beyond a fixed value. The proof is based on two-dimensional potential theory. This is joint work with Elliott Lieb and Nicolas Rougerie.

Dimitri Zvonkine *Laughlin states and vector bundles on moduli spaces of curves*

Abstract : We explain the relationship between the Laughlin states in fractional quantum Hall effect and the vector bundle of theta-functions on the moduli space M_g of genus g curves. We also show how to compute the Chern classes of the latter vector bundle. This is joint work in progress with Semyon Klevtsov.